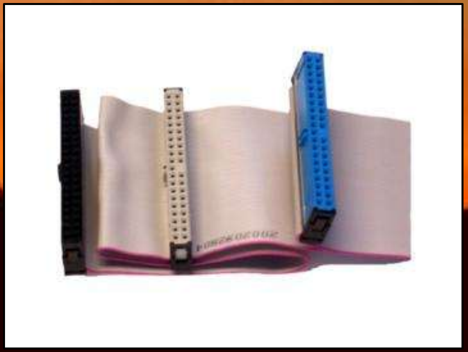
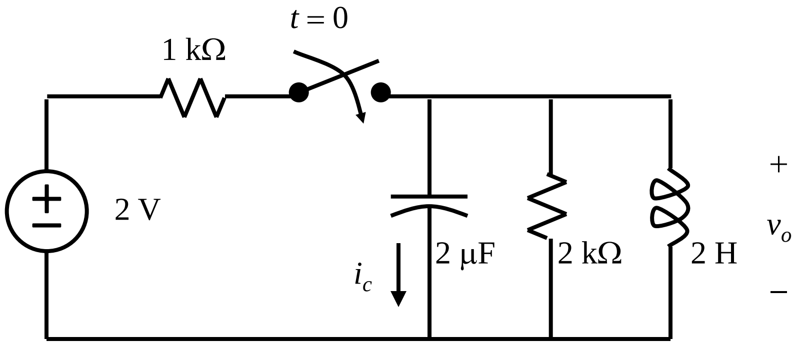
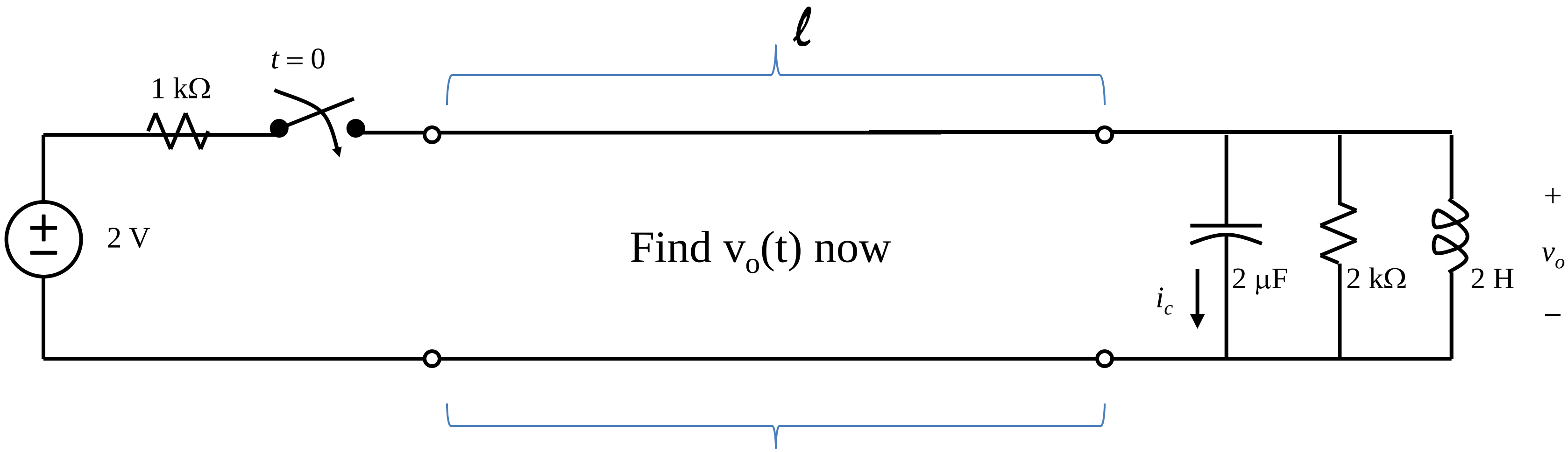
Transmission Lines



Lecture 3



Find $v_o(t)$



What if this is 25 km ???

It's a T-line if:

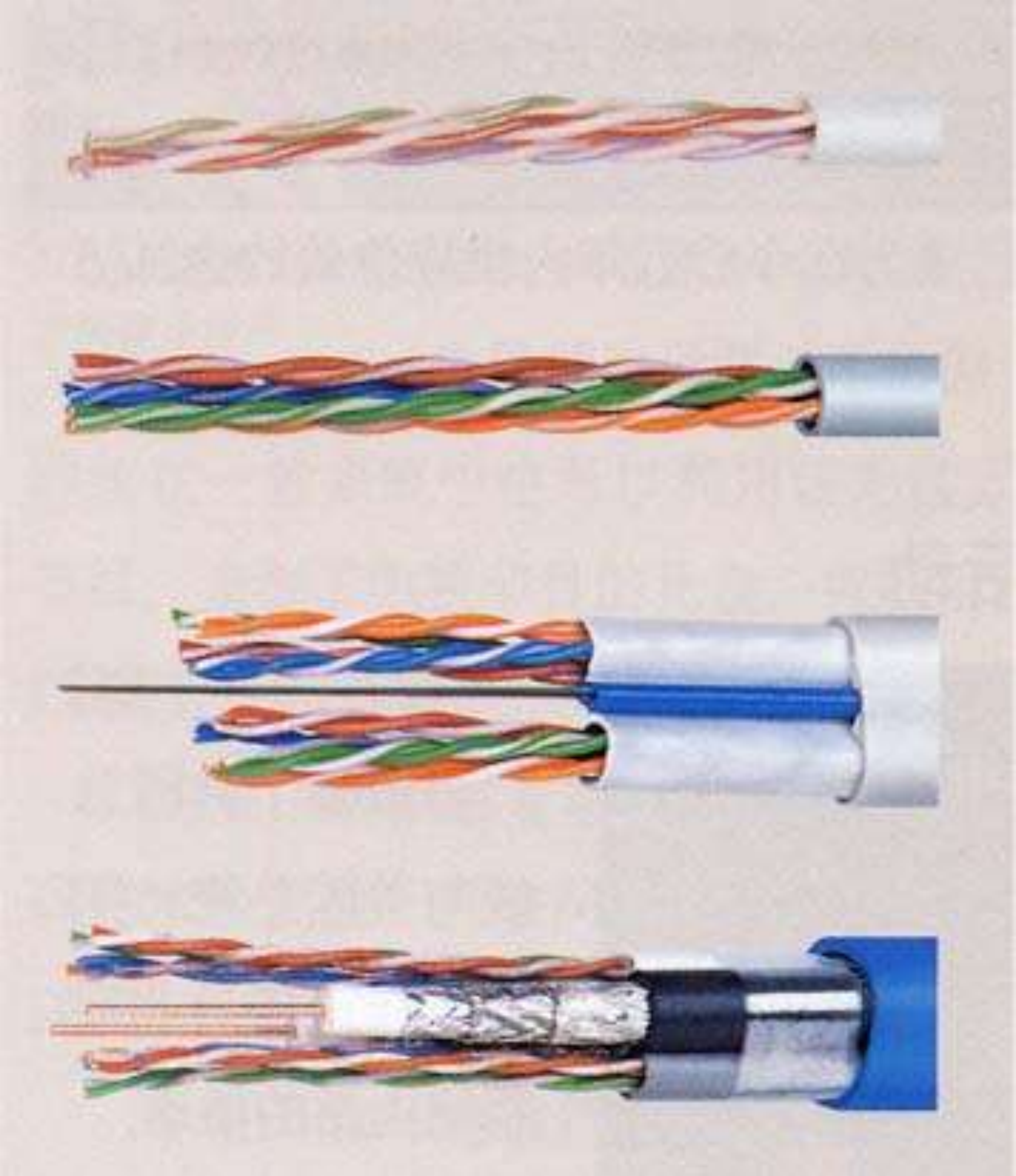
$$\frac{\ell}{\lambda} \sim \frac{f\ell}{c} \gtrsim 1\%$$

$\Rightarrow$ 3 MHz ~ m's

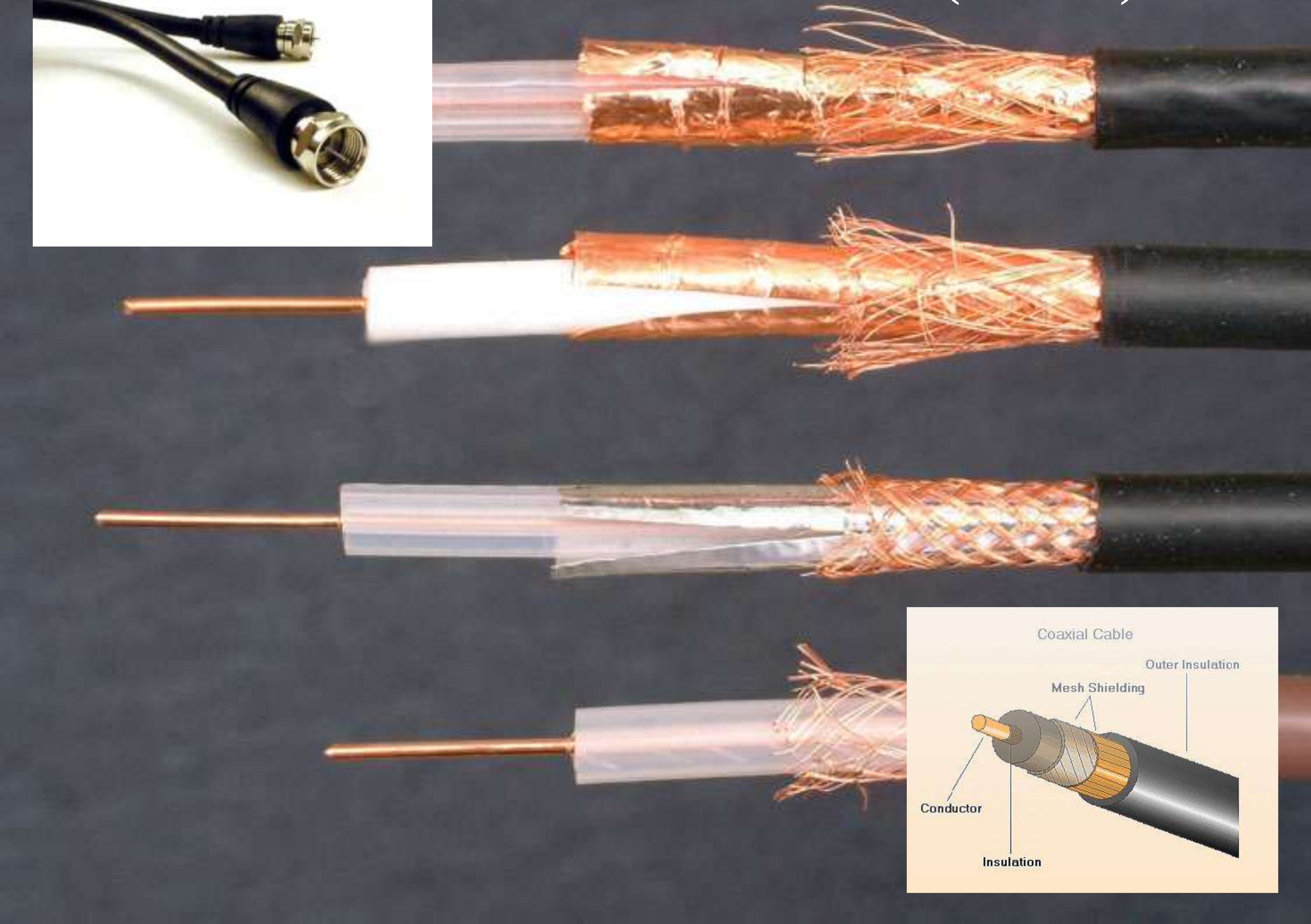
3 GHz ~ mm's

One place where E&M meets circuits

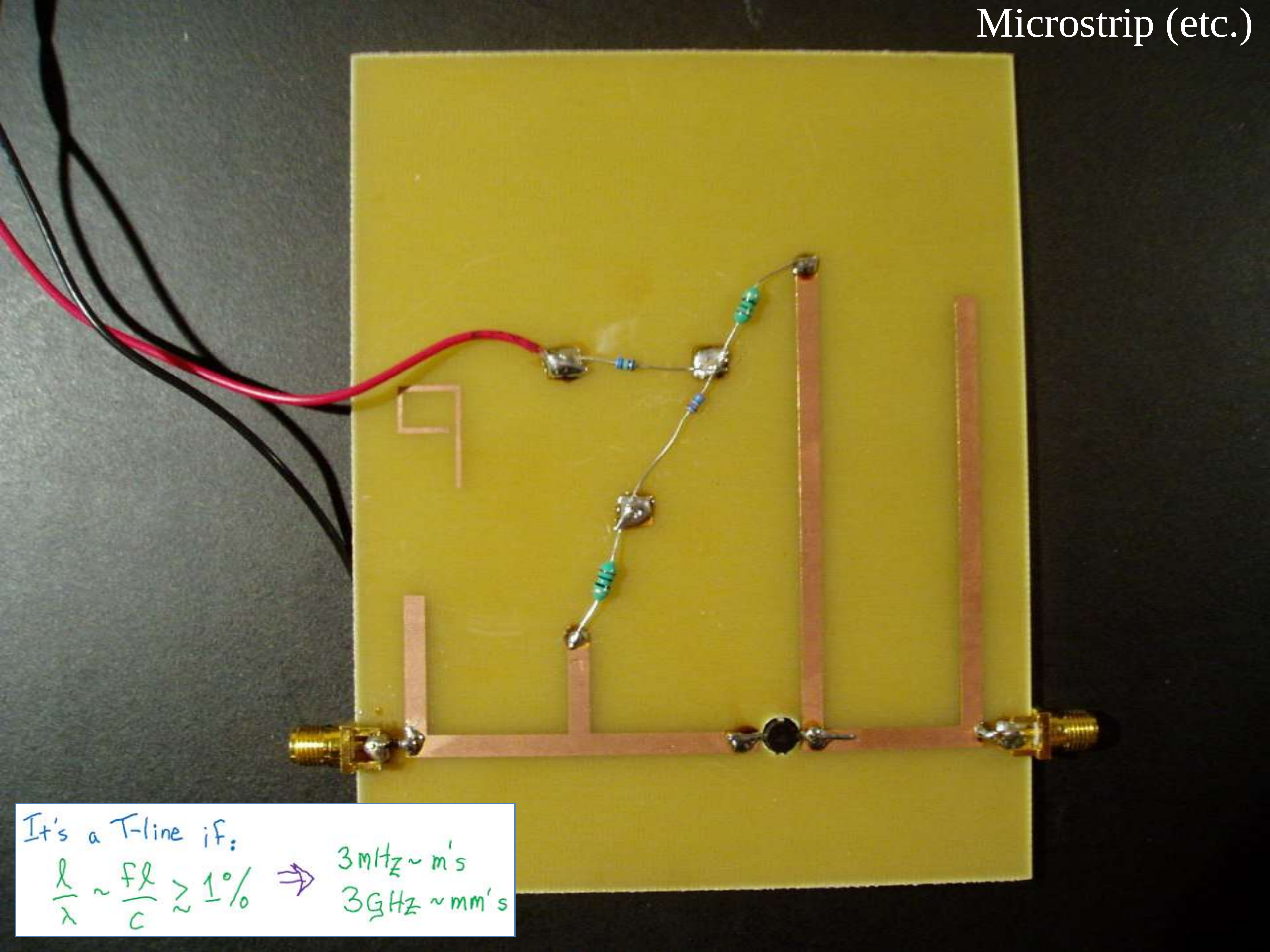
Twisted Pair



Coaxial (Co-Ax) Cable



Microstrip (etc.)



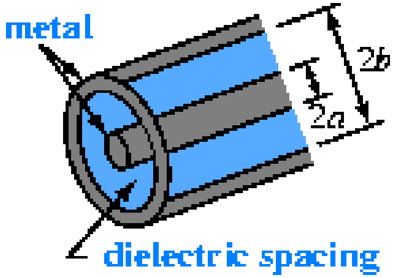
It's a T-line if:

$$\frac{\ell}{\lambda} \sim \frac{f\ell}{c} \gtrsim 1\% \Rightarrow$$

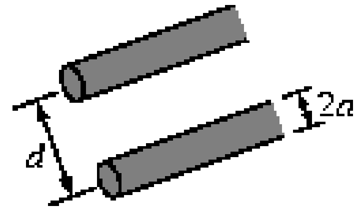
3 MHz ~ m's

3 GHz ~ mm's

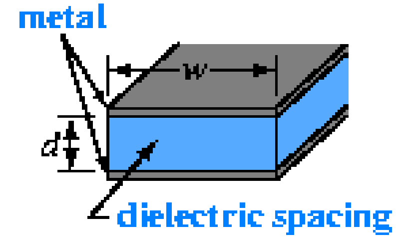
Waveguides



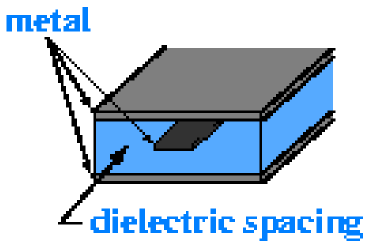
(a) Coaxial line



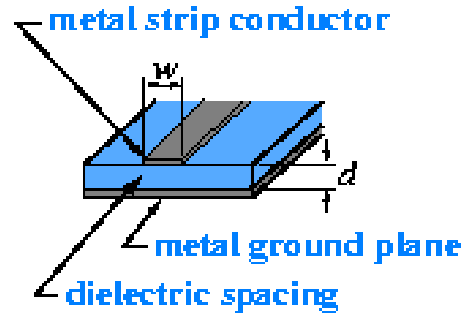
(b) Two-wire line



(c) Parallel-plate line

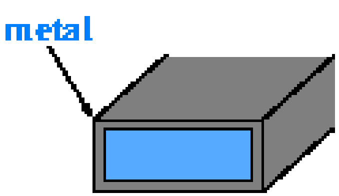


(d) Strip line

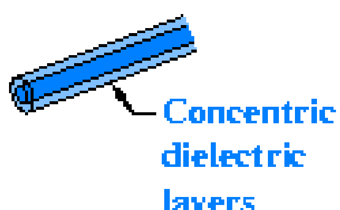


(e) Microstrip line

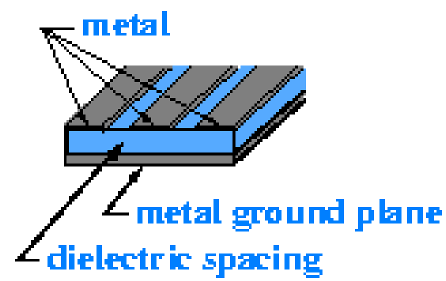
Transverse Electro Magnetic → **TEM Transmission Lines**



(f) Rectangular waveguide



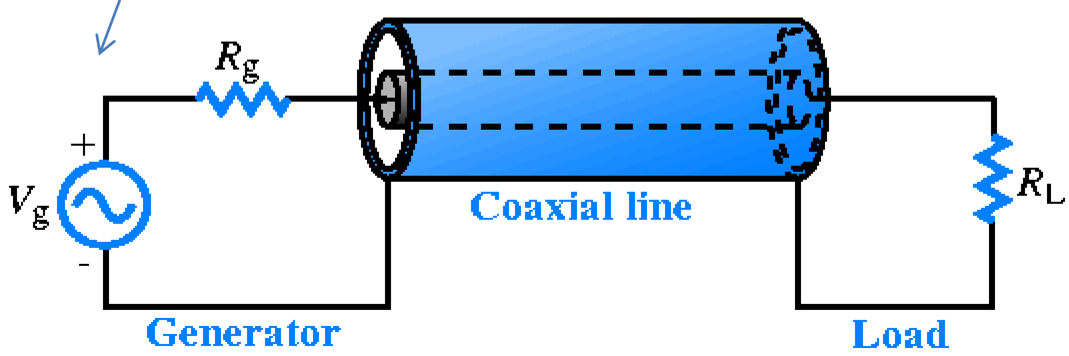
(g) Optical fiber

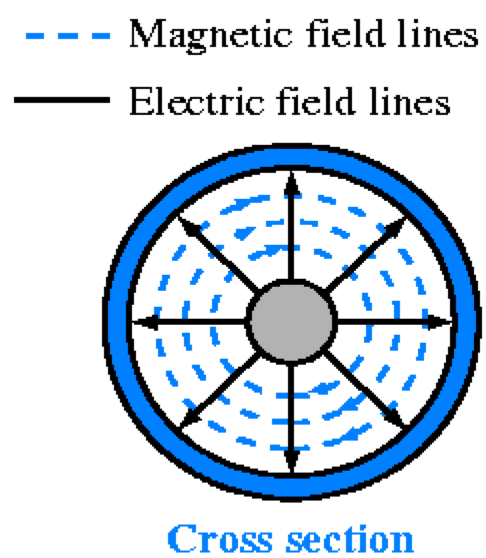


(h) Coplanar waveguide

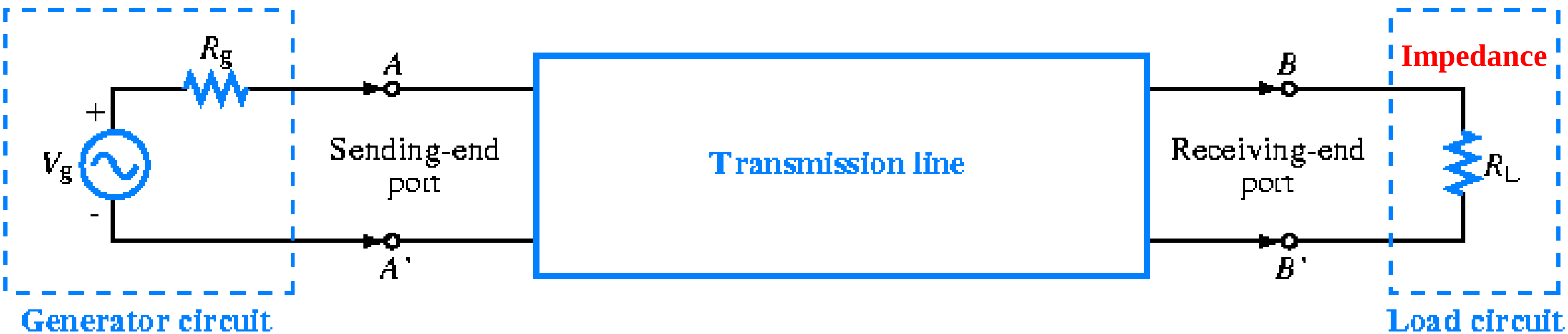
Higher Order Transmission Lines

If it's AC, then can use *phasors* …





Thévenin Equivalent

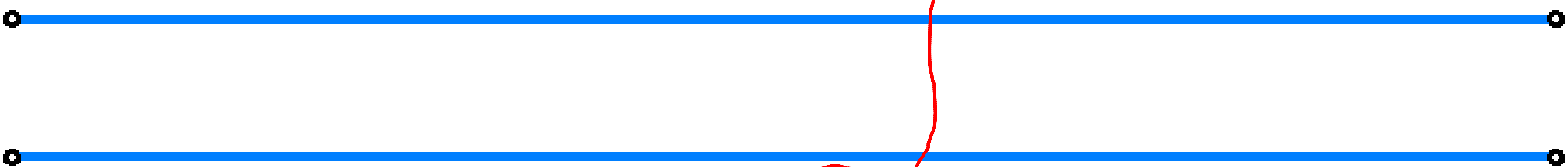


$V_{AA'} \neq V_{BB'}$

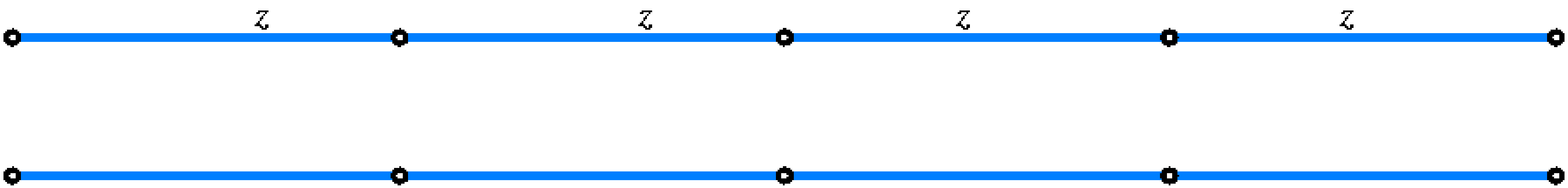
(phase delay)

$\Rightarrow$ in addition to phase delay, T-lines exhibit power loss, dispersion & reflections.

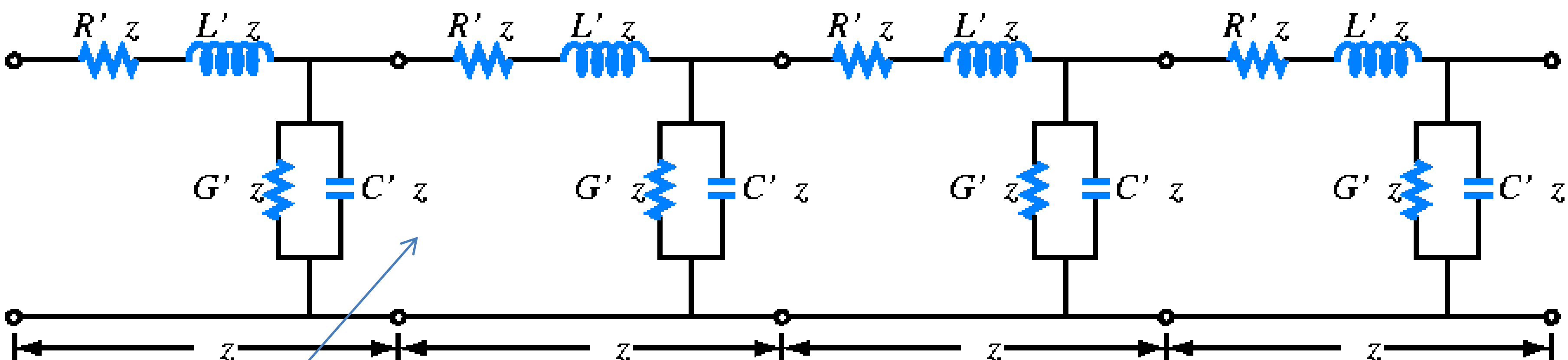
Kinda Like or



(a) Parallel-wire representation



(b) Differential sections each z long

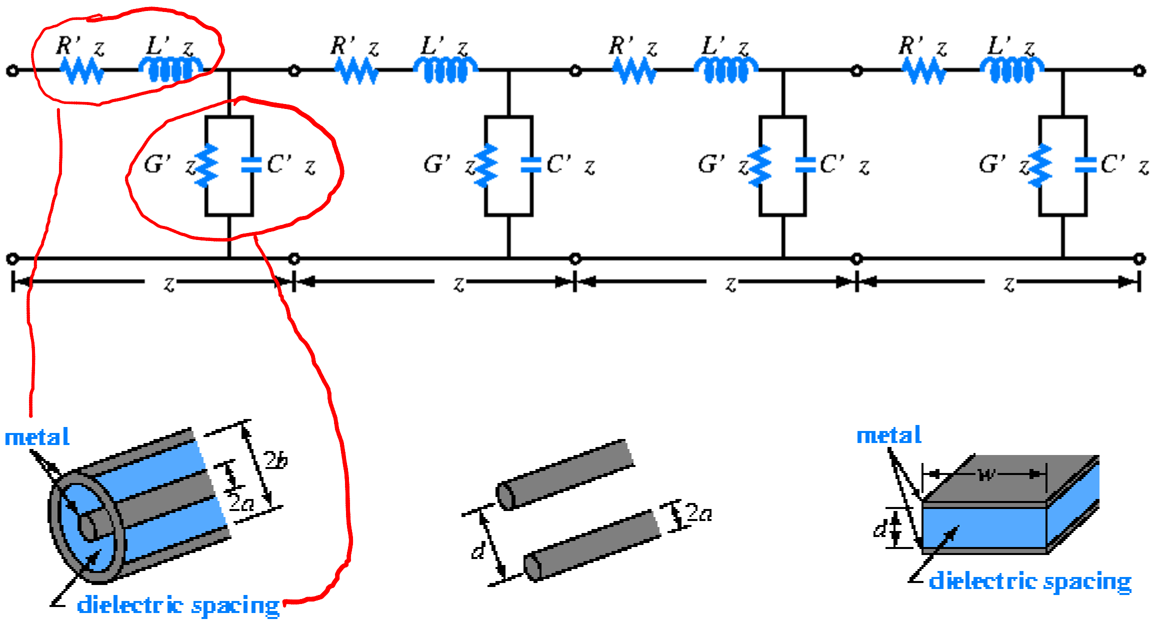


(c) Each section is represented by an equivalent circuit

(Lumped-element circuit model)

…*kinda* **like Thévenin** ☺

NOTE: not the only model, but the math is all the same



(a) Coaxial line (b) Two-wire line (c) Parallel-plate line

T-Line Parameters

R'	Ω/m	
L'	H/m	Differential (per unit Length)
G'	S/m	
C'	F/m	

Parameter	Coaxial	Two Wire	Parallel Plate	Unit
R'	$\dfrac{R_s}{2\pi}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$	$\dfrac{R_s}{\pi a}$	$\dfrac{2R_s}{w}$	Ω/m
L'	$\dfrac{\mu}{2\pi}\ln(b/a)$	$\dfrac{\mu}{\pi}\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]$	$\dfrac{\mu d}{w}$	H/m
G'	$\dfrac{2\pi\sigma}{\ln(b/a)}$	$\dfrac{\pi\sigma}{\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]}$	$\dfrac{\sigma w}{d}$	S/m
C'	$\dfrac{2\pi\varepsilon}{\ln(b/a)}$	$\dfrac{\pi\varepsilon}{\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]}$	$\dfrac{\varepsilon w}{d}$	F/m

Notes: (1) Refer to Fig. 2-4 for definitions of dimensions. (2) μ, ε, and σ pertain to the insulating material between the conductors. (3) $R_s = \sqrt{\pi f \mu_c/\sigma_c}$. (4) μ_c and σ_c pertain to the conductors. (5) If $(d/2a)^2 \gg 1$, then $\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right] \simeq \ln(d/a)$.

- We can (and will) calculate these... (for now, plug & chug)
- ϵ, μ, and σ are measured, not derived...
- T-Line properties are frequency dependent!

Exercise 2.1 Use Table 2-1 to compute the line parameters of a two-wire air line whose wires are separated by a distance of 2 cm, and each is 1 mm in radius. The wires may be treated as perfect conductors with $\sigma_c = \infty$.

Solution: Two-wire air line: Because medium between wires is air, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$ and $\sigma = 0$.

$\epsilon_0 \cong 8.85 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ H/m

$$d = 2 \text{ cm}, \qquad a = 1 \text{ mm}, \qquad \sigma_c = \infty$$

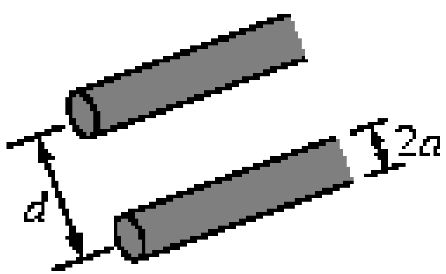
$$R_s = \left[\frac{\pi f \mu_c}{\sigma_c}\right]^{1/2} = 0$$

$$R' = 0$$

$$L' = \frac{\mu_0}{\pi} \ln\left[\left(\frac{d}{2a}\right) + \sqrt{\left(\frac{d}{2a}\right)^2 - 1}\right]$$

$$= \left(\frac{4\pi \times 10^{-7}}{\pi}\right) \ln\left[\left(\frac{20}{2}\right) + \sqrt{\left(\frac{20}{2}\right)^2 - 1}\right]$$

$$= 4 \times 10^{-7} \ln[10 + \sqrt{99}] = 1.2 \quad (\mu\text{H/m}).$$

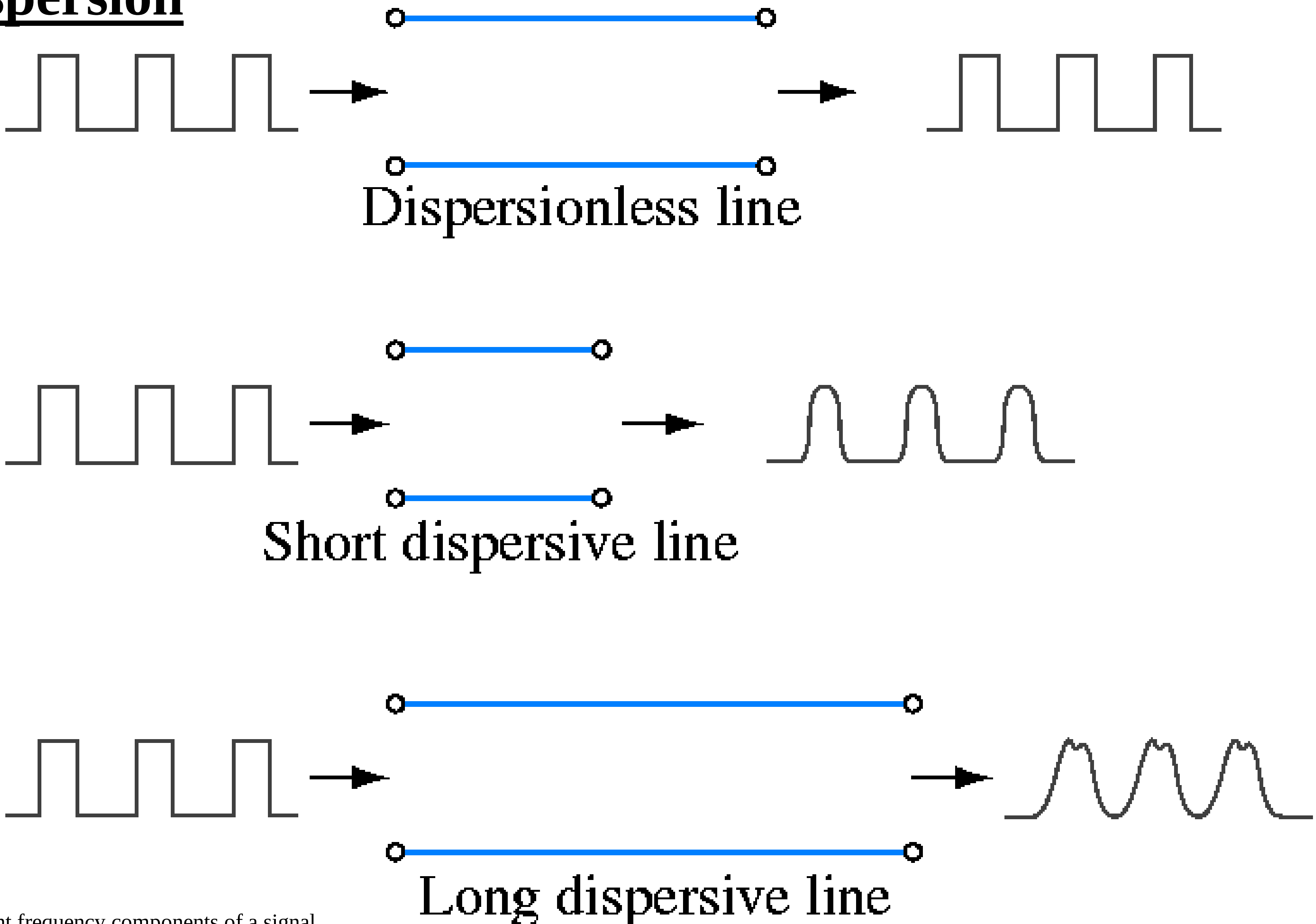


$$G' = 0 \qquad \text{because } \sigma = 0$$

$$C' = \frac{\pi \varepsilon_0}{\ln\left[\left(\frac{d}{2a}\right) + \sqrt{\left(\frac{d}{2a}\right)^2 - 1}\right]} = \frac{\pi \times 8.85 \times 10^{-12}}{\ln[10 + \sqrt{99}]} = 9.29 \quad (\text{pF/m}).$$

Power Loss

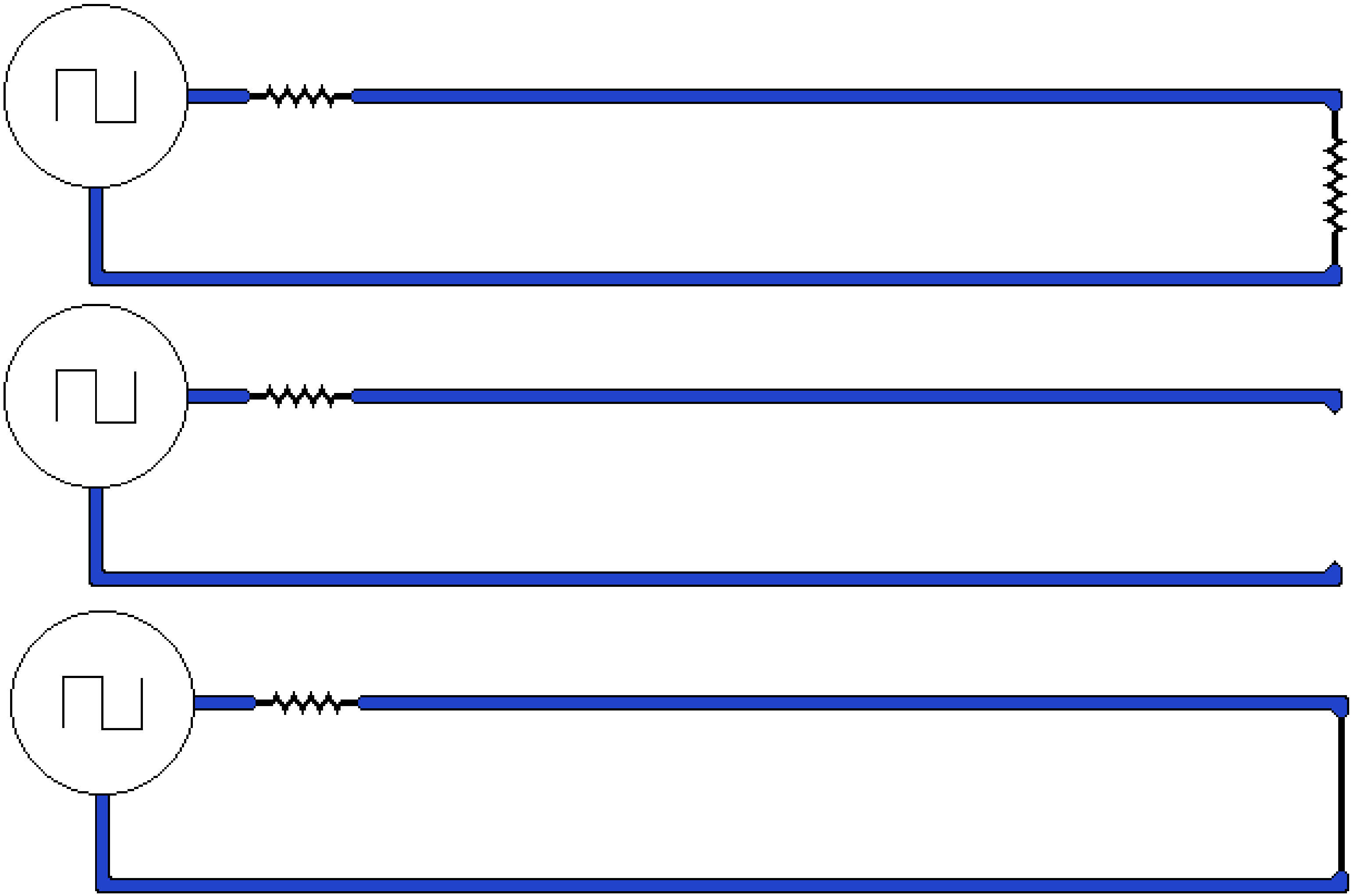
Dispersion*

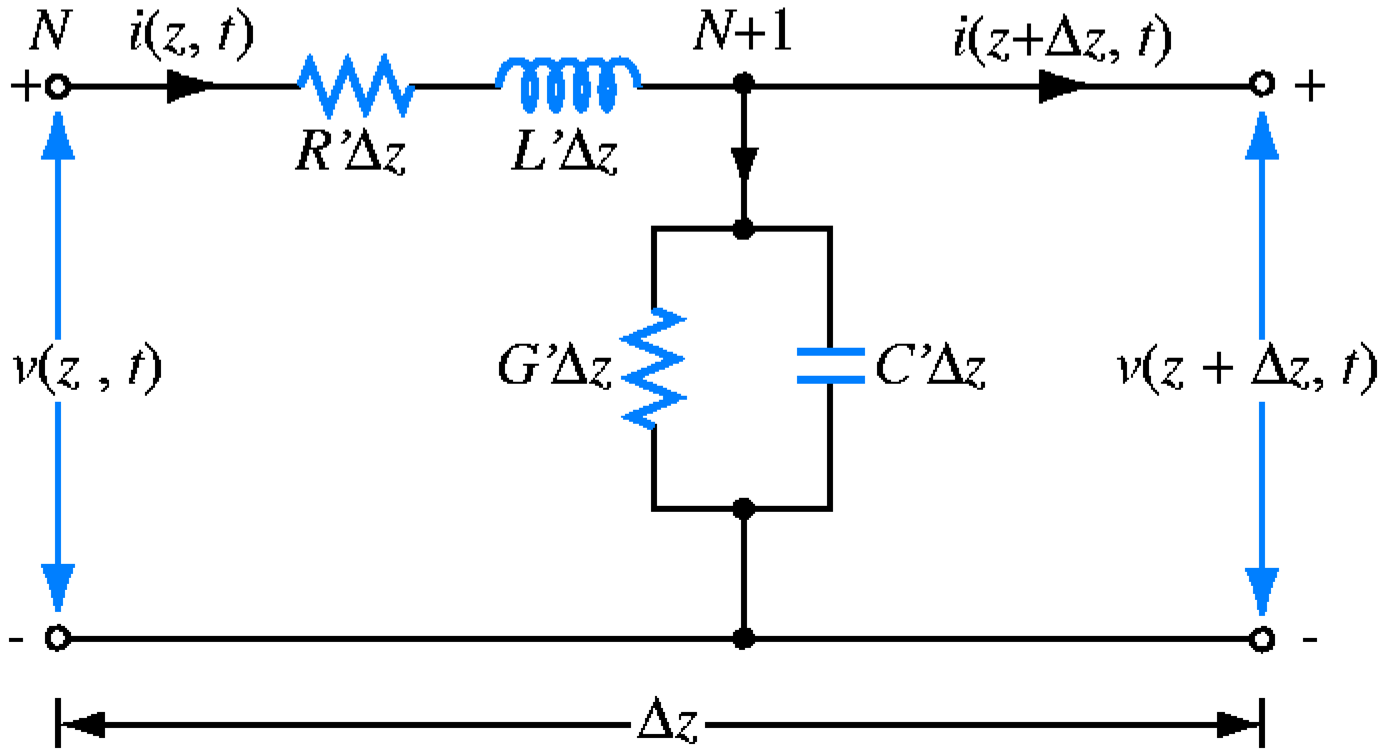


* Different frequency components of a signal travel at different speeds and undergo different attenuations as they travel down the line

Hmmm… it's not AC, so how can we use phasors ???

Reflection




N
i(z, t)
N+1
i(z+Δz, t)
+
+
R'Δz
L'Δz
v(z , t)
G'Δz
C'Δz
v(z + Δz, t)
-
-
Δz

Lloyd Espenschied and Herman A. Affel*, [ca. 1949]

* Filed 1st patent for coaxial cable [1929]

- Transmission lines transmit energy and signals from a source (generator) to a load
- Distinguishing characteristics of a transmission line:

  \- the devices to be connected are separated by distances comparable to or larger than the signal wavelength

  \- the parameters of the circuit are distributed and are evaluated on a per-unit length basis:

  R' – resistance per unit length, Ω/m

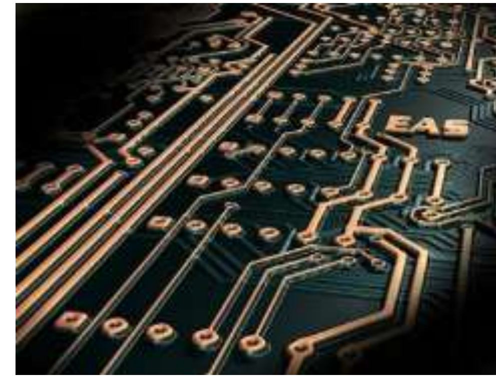
  L' – inductance per unit length, H/m

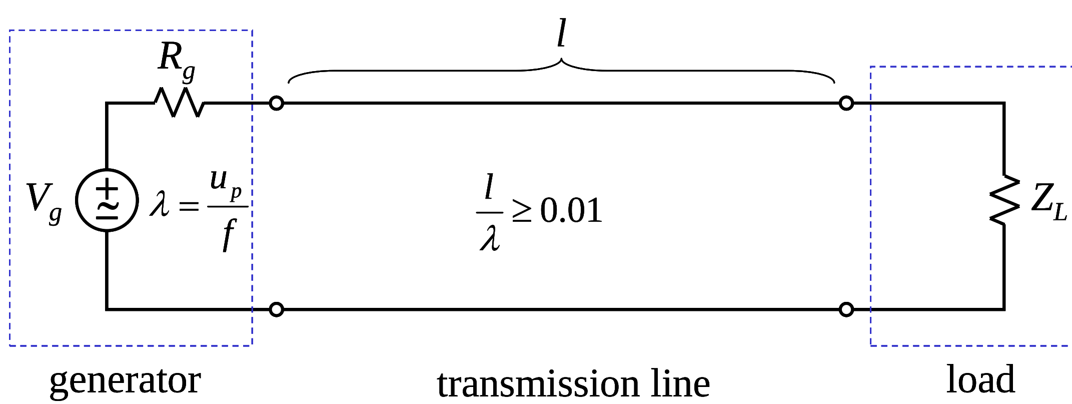
  G' – conductance per unit length, S/m

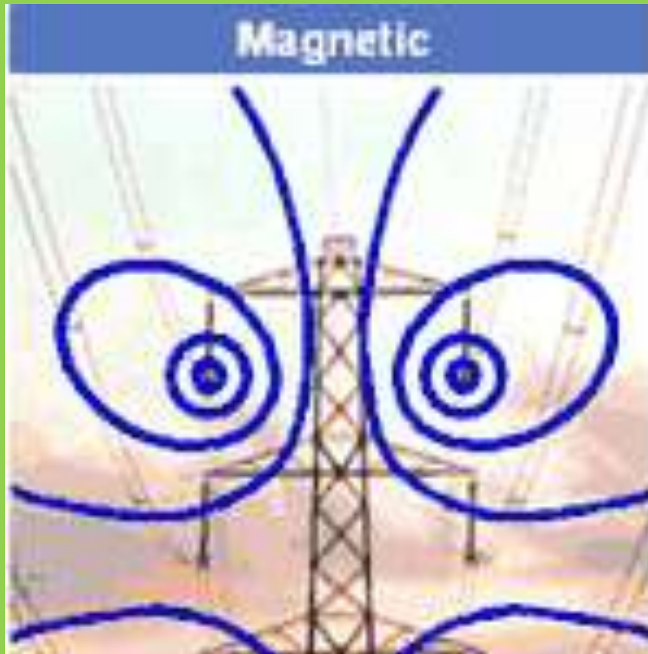
  C' – capacitance per unit length, F/m

  \- transmission lines are circuit elements that have complex impedances, which are functions of line length and signal frequency

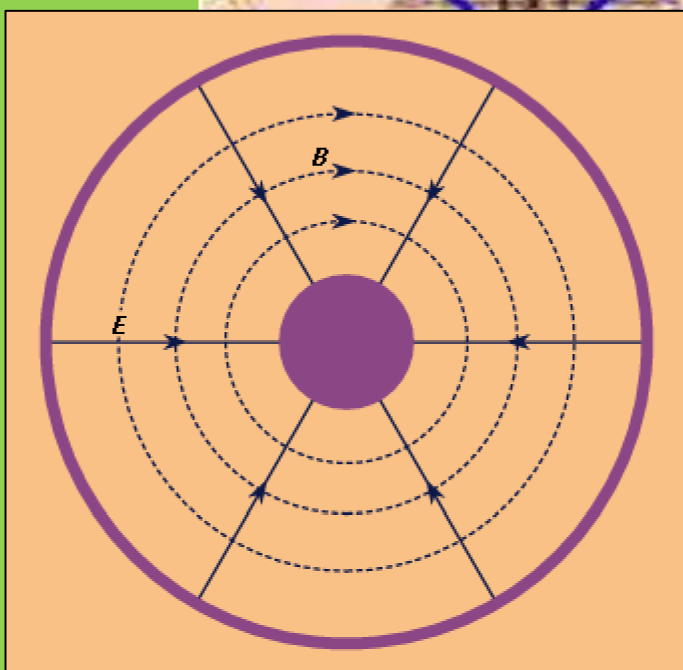
- TEM transmission lines
  - two-wire line
  - coaxial cable
  - parallel-plate line

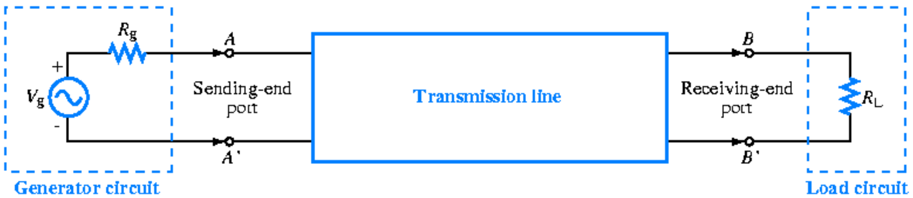


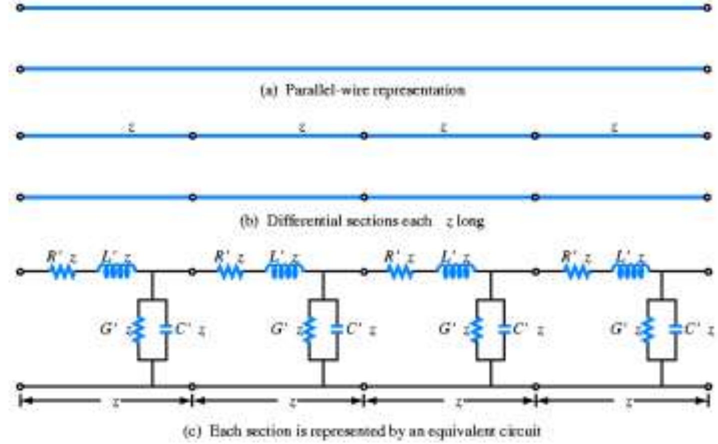



Magnetic


Electric

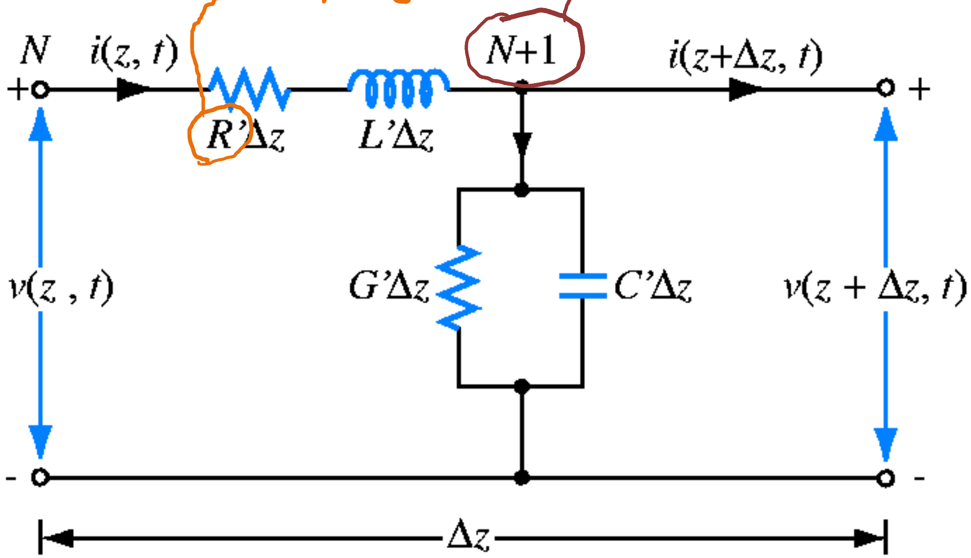

B
E



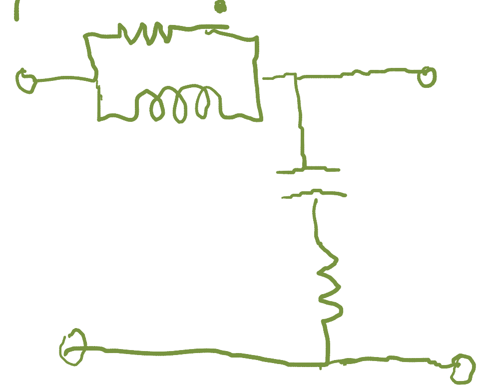


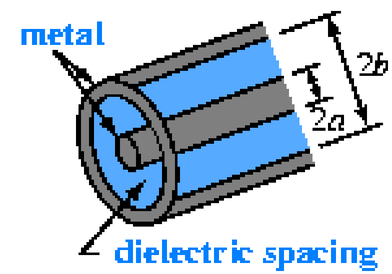
How many? What if T-Line is 1m Long?

per unit Length

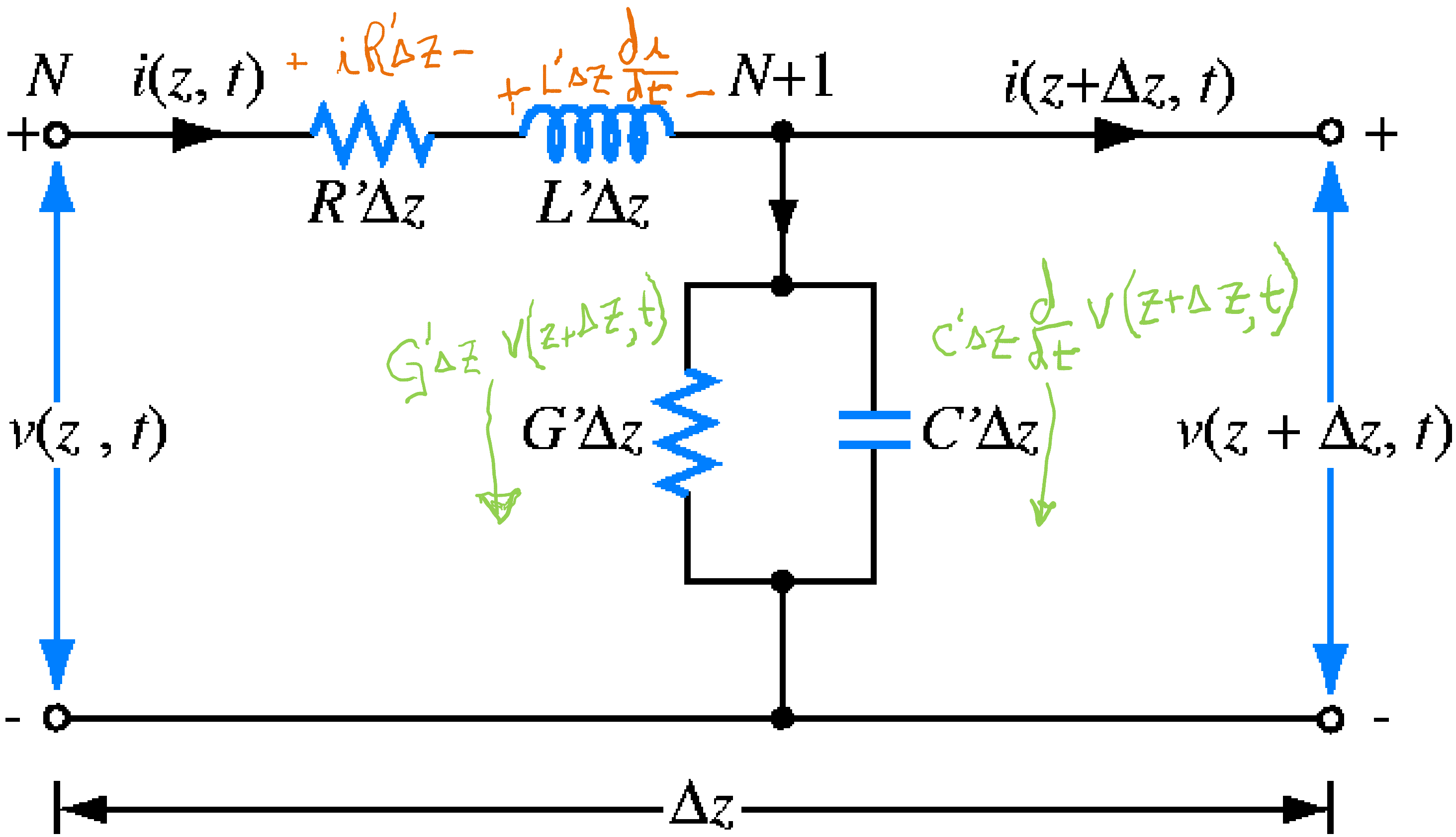


Why NOT ?



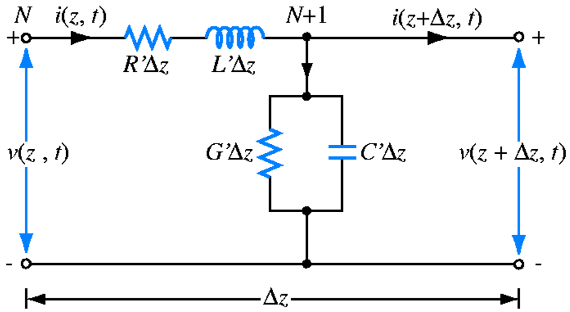


Kirchoff Time!



$$v(z,t) - i(z,t)R'\Delta z - L'\Delta z \frac{d}{dt} i(z,t) = v(z+\Delta z, t) \qquad \text{KVL}$$

$$i(z,t) - G'\Delta z\, v(z+\Delta z,t) - C'\Delta z \frac{d}{dt} v(z+\Delta z,t) = i(z+\Delta z,t) \qquad \text{KCL}$$



Now, $\dfrac{v(z+\Delta z,t)-v(z,t)}{\Delta z} \rightarrow \dfrac{d}{dz}v(z,t)$

as $\Delta z \rightarrow 0$

so, our KVL & KCL eqtn. become $\Rightarrow$

$$-\frac{dv(z,t)}{dz} = R'\,i(z,t) + L'\frac{d\,i(z,t)}{dt}$$

$$-\frac{d\,i(z,t)}{dz} = G'v(z,t) + C'\frac{dv(z,t)}{dt}$$

coupled equations . . .

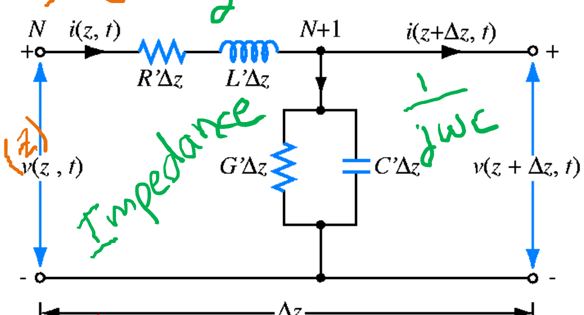
Query: why **do** we do derivations?? ☺

Transmission Line Equations
(a.k.a. Telegrapher's Equations)

Phasors *

$\tilde{I}(z)$

$j\omega L$

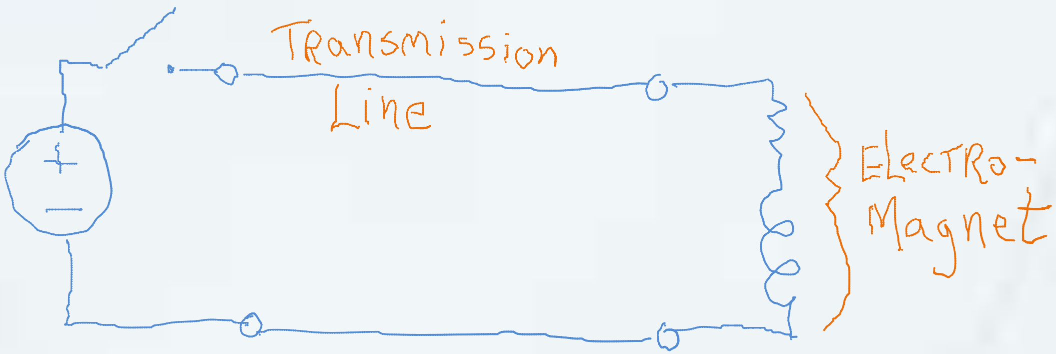


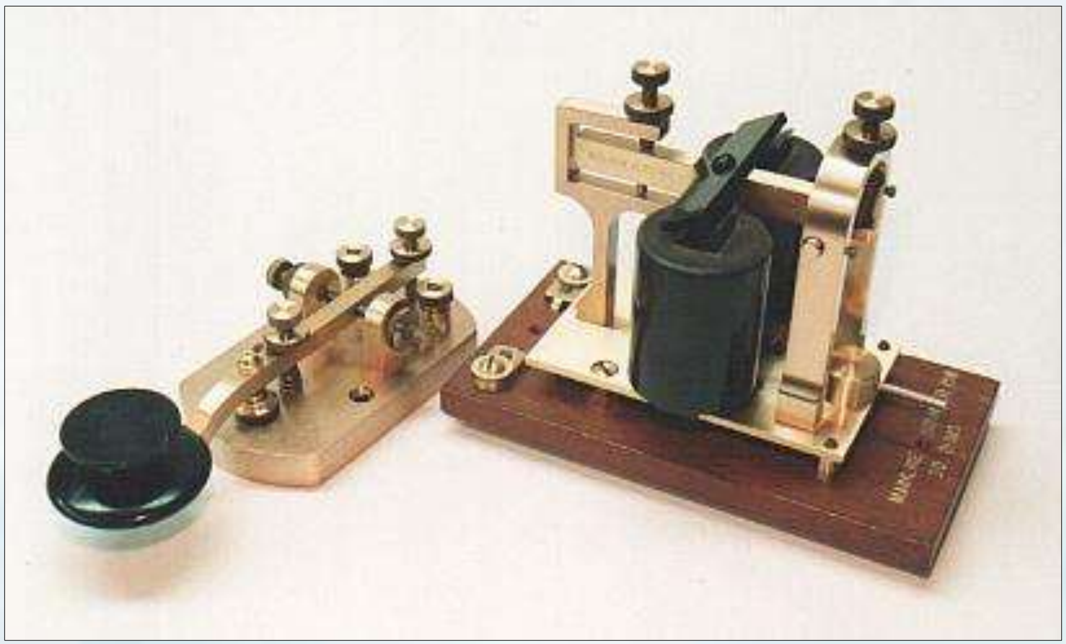
$\tilde{V}(z)$ Impedance $\frac{1}{j\omega C}$ $\Rightarrow$

$$-\frac{d\tilde{V}(z)}{dz} = (R'+j\omega L')\,\tilde{I}(z)$$

$$-\frac{d\tilde{I}(z)}{dz} = (G'+j\omega C')\,\tilde{V}(z)$$

* why?. Is it practical ?

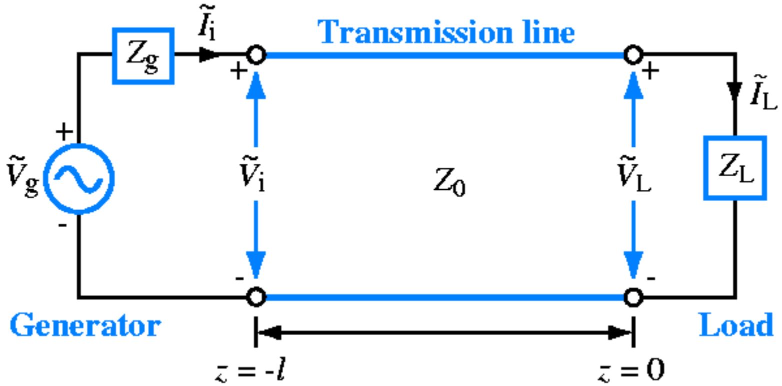




$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\,\tilde{I}(z)$$

$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\,\tilde{V}(z)$$

The voltage and the current are <u>inter</u>-related!



what assumption?

$$-\frac{d^2\tilde{V}(z)}{dz^2} = (R'+j\omega L')\frac{d\tilde{I}(z)}{dz}$$

$$-\frac{d\tilde{I}(z)}{dz} = (G'+j\omega C')\tilde{V}(z)$$

$$\Rightarrow \quad \frac{d^2\tilde{V}(z)}{dz^2} = \gamma^2\tilde{V}(z)$$

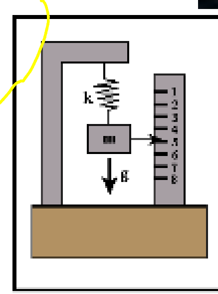
Wave EQTN.

with $\gamma \equiv \sqrt{(R'+j\omega L')(G'+j\omega C')} = \alpha + j\beta$

propagation constant

can likewise show $\Rightarrow$

$$\frac{d^2\tilde{I}(z)}{dz^2} = \gamma^2\tilde{I}(z)$$



$$\frac{d^2\tilde{V}(z)}{dz^2} = \gamma^2\,\tilde{V}(z)$$

$$\frac{d^2\tilde{I}(z)}{dz^2} = \gamma^2\,\tilde{I}(z)$$

$$\gamma \equiv \sqrt{(R'+j\omega L')(G'+j\omega C')} = \alpha + j\beta = \gamma$$

$$\xrightarrow[\text{differential equation}]{\text{solve}}$$

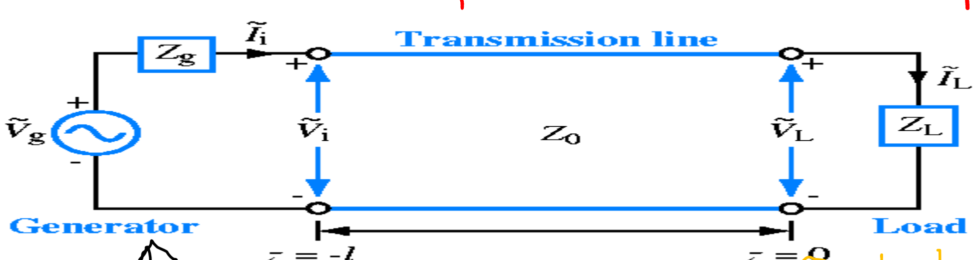
$$\underbrace{\begin{aligned}\tilde{V}(z) &= V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}\\ \tilde{I}(z) &= I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}\end{aligned}}_{\text{Plug 'em back in to verify!}}$$

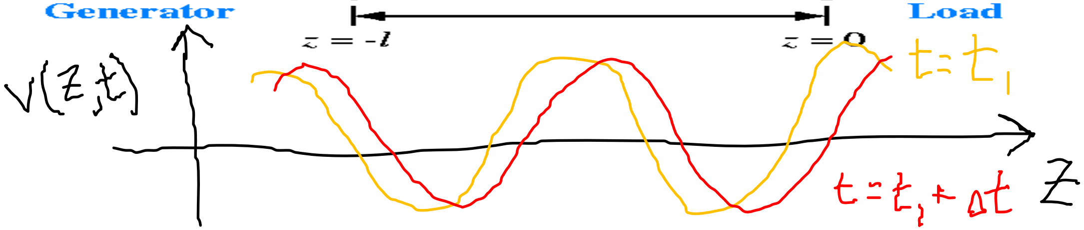
(are there any other solvs?)

Hmmm...

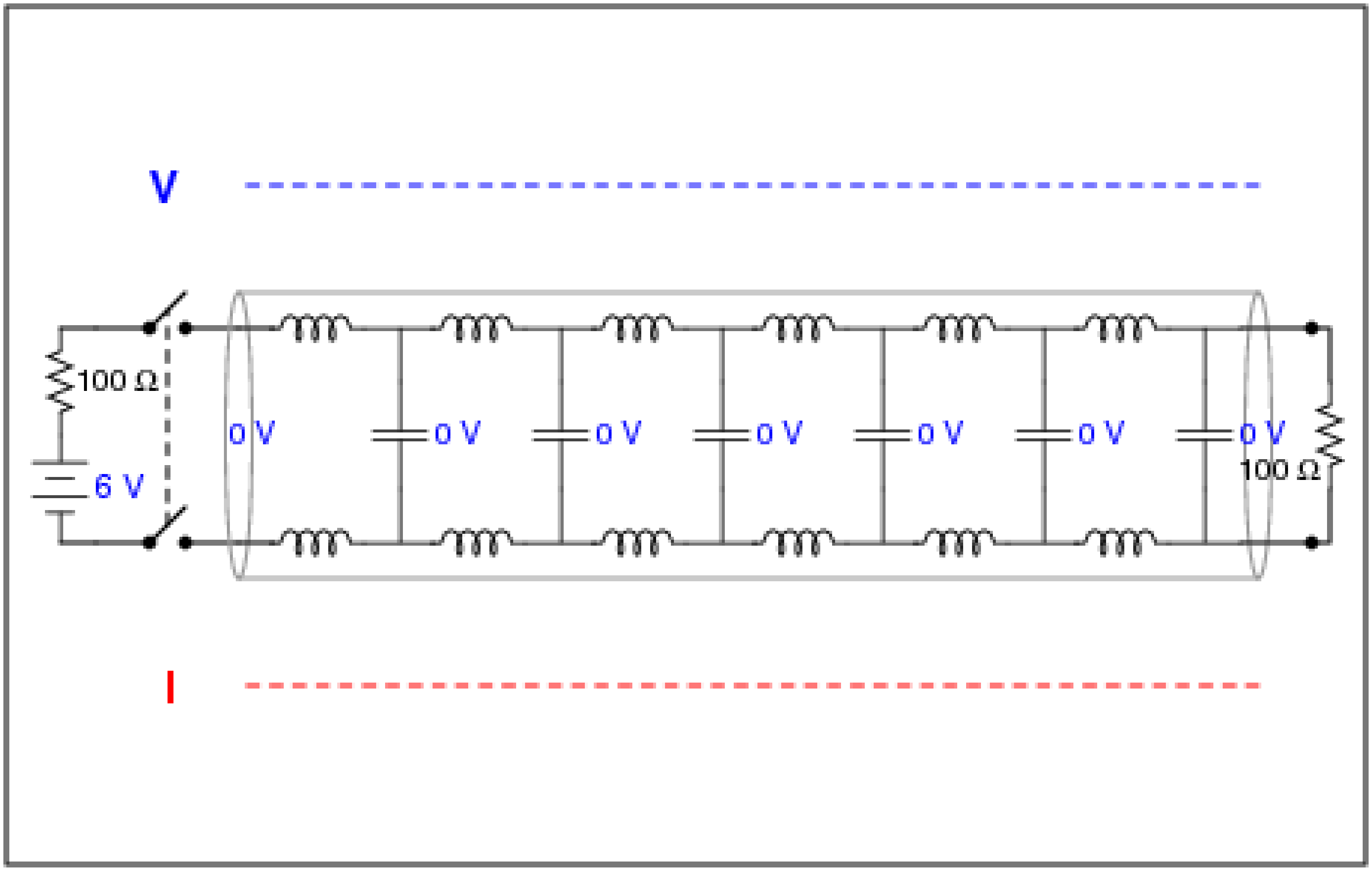
Let's look at the 1st term of voltage

$$\Rightarrow \underbrace{\tilde{V}(z) = V_0^+ e^{-\alpha z} e^{-j\beta z}}_{\text{phasor}} \rightarrow \underbrace{v(z,t) = V_0^+ e^{-\alpha z}\cos(\omega t - \beta z)}_{\text{time domain}}$$





propagation in +z direction



V
100 Ω
6 V
0 V
0 V
0 V
0 V
0 V
0 V
0 V
100 Ω
I

Wave Propagation on T-Lines

Wave in +Z direction

Wave in −Z direction

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

can have both at once ... they add by superposition

$$\tilde{V}(z) = V_0^+ e^{-\alpha z} e^{-j\beta z} \Rightarrow v(z,t) = V_0^+ \underbrace{e^{-\alpha z}} \underbrace{\cos(\omega t - \beta z)}_{\phi}$$

attenuation

$$\alpha \equiv \mathrm{Re}\{\gamma\}$$

speed

$$\beta \equiv \mathrm{Im}\{\gamma\}$$

phase velocity

$$u_p = \frac{\omega}{\beta} = f\lambda$$

so $\beta = 2\pi/\lambda$

It's a T-line if:

$$\frac{\ell}{\lambda} \sim \frac{f\ell}{c} \gtrsim 1\% \Rightarrow$$

3 MHz ~ m's

3 GHz ~ mm's

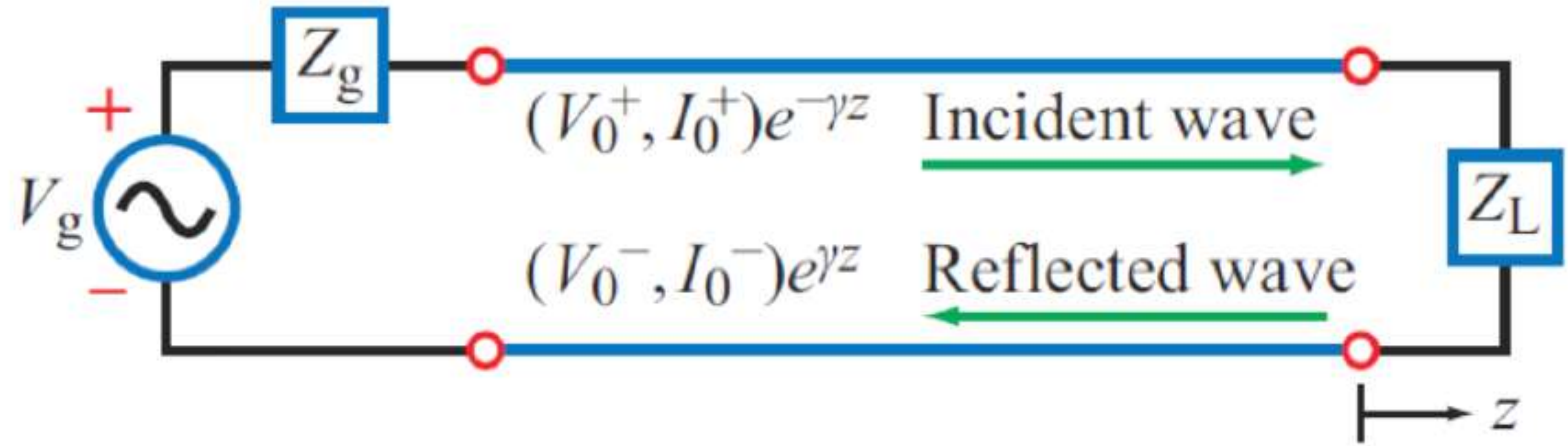


Figure 2-9: In general, a transmission line can support two traveling waves, an incident wave [with voltage and current amplitudes (V_0^+, I_0^+)] traveling along the $+z$-direction (towards the load) and a reflected wave [with (V_0^-, I_0^-)] traveling along the $-z$-direction (towards the source).

Characteristic Impedance

$$(R'+j\omega L')\times$$

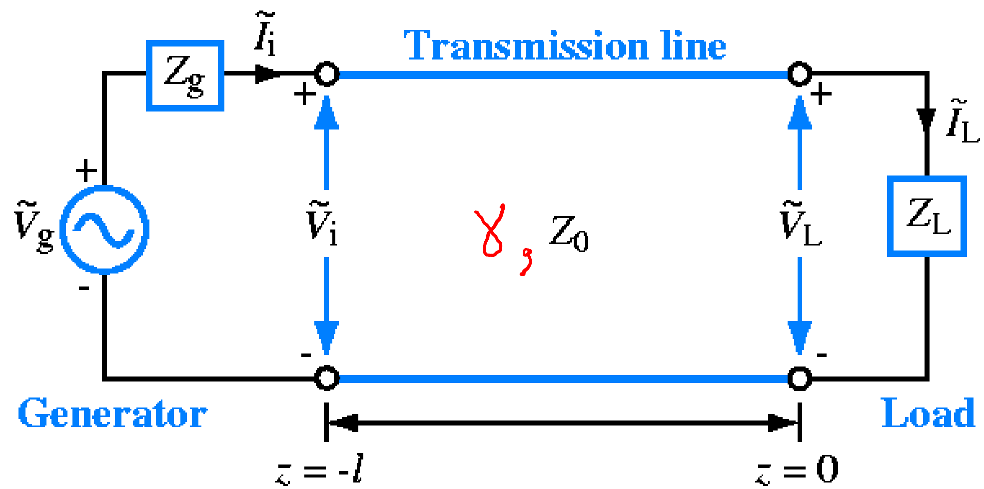
$$-\frac{d}{dz}\tilde{V}(z) = -V_0^+\frac{d}{dz}e^{-\gamma z} + -V_0^-\frac{d}{dz}e^{+\gamma z} = \frac{-d\tilde{V}(z)}{dz} = (R'+j\omega L')\,\tilde{I}(z) = (R'+j\omega L')\,\tilde{I}(z) = I_0^+e^{-\gamma z} + I_0^-e^{+\gamma z}$$

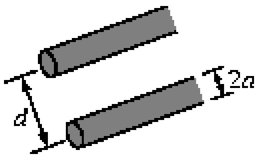
$$\Rightarrow \quad \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = \sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}} \doteq Z_0$$

Characteristic Impedance of Transmission Line

(not a function of z or t)

(not equal to $\tilde{V}/\tilde{I}$)





Exercise 2.4 A two-wire air line has the following line parameters: $R' = 0.404$ (mΩ/m), $L' = 2.0$ (μH/m), $G' = 0$, and $C' = 5.56$ (pF/m). For operation at 5 kHz, determine (a) the attenuation constant α, (b) the phase constant β, (c) the phase velocity u_p, and (d) the characteristic impedance Z_0.

Solution: Given:

$$R' = 0.404 \text{ (m}\Omega\text{/m)}, \qquad G' = 0,$$
$$L' = 2.0 \text{ (}\mu\text{H/m)}, \qquad C' = 5.56 \text{ (pF/m)}.$$

(a)

$$\alpha = \mathfrak{Re}\left\{[\underbrace{(R' + j\omega L')(G' + j\omega C')}_{\gamma}]^{1/2}\right\}$$

Oops! Forgot the 2π! $(\omega = 2\pi f)$

$$= \mathfrak{Re}\left\{[(0.404 \times 10^{-3} + j5 \times 10^3 \times 2 \times 10^{-6})(0 + j5 \times 10^3 \times 5.56 \times 10^{-12})]^{1/2}\right\}$$
$$= \mathfrak{Re}[3.37 \times 10^{-7} + j1.05 \times 10^{-4}]$$
$$\alpha = 3.37 \times 10^{-7} \quad \text{(Np/m)}.$$

Note: final numerical answers are fine though ☺

(b) From part (a),

$$\beta = \mathfrak{Im}\left\{[(R' + j\omega L')(G' + j\omega C')]^{1/2}\right\}$$
$$= 1.05 \times 10^{-4} \quad \text{(rad/m)}.$$

(c) $$u_p = \frac{\omega}{\beta} = \frac{2\pi \times 5 \times 10^3}{\cancel{1.05 \times 10^{-4}}} = 3 \times 10^8 \quad \text{(m/s)}.$$

(d) $$Z_0 = \frac{R' + j\omega L'}{\alpha + j\beta}$$

an equiv. form for Z_0

$$= \frac{0.404 \times 10^{-3} + j5 \times 10^3 \times 2 \times 10^{-6}}{3.37 \times 10^{-7} + j1.05 \times 10^{-4}}$$
$$= (600 - j2)\ \Omega.$$

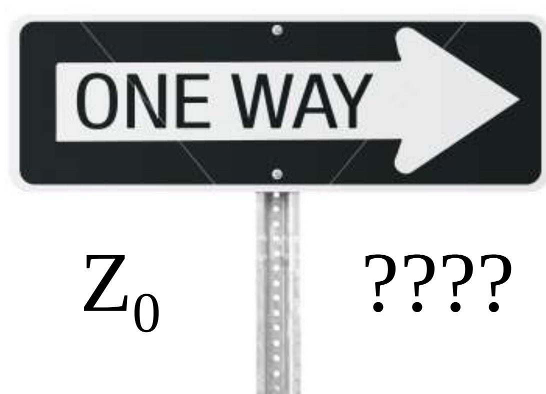
Table 3. Comparing $|Z_o|$ for a variety of transmission lines at 14 MHz.[3]

		Zo| (Ohms)
Frequency (MHz)		14
24AWG	2 twists	45
	3 twists	44
	4 twists	43
	5 twists	41
26AWG	2 twists	57
	3 twists	54
	4 twists	48
	5 twists	47
28AWG	3 twists	51
	4 twists	49
	5 twists	47
30AWG	3 twists	49
	4 twists	46
	5 twists	47

For coaxial cables the characteristic impedance will be typically between 20 and 150 ohms (the ones you use in Lab are 50 ohms).

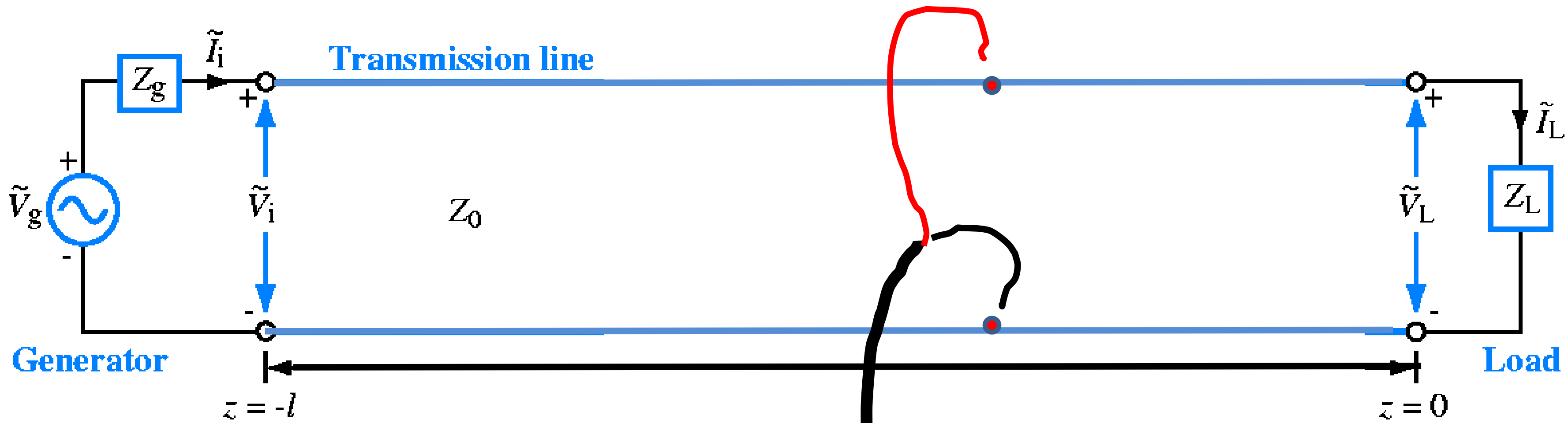
The length of the cable makes no difference whatsoever in regard to the characteristic impedance.

+ = **50 ohms ?????**



Z_0 ????

Example of acoustic "transmission lines"

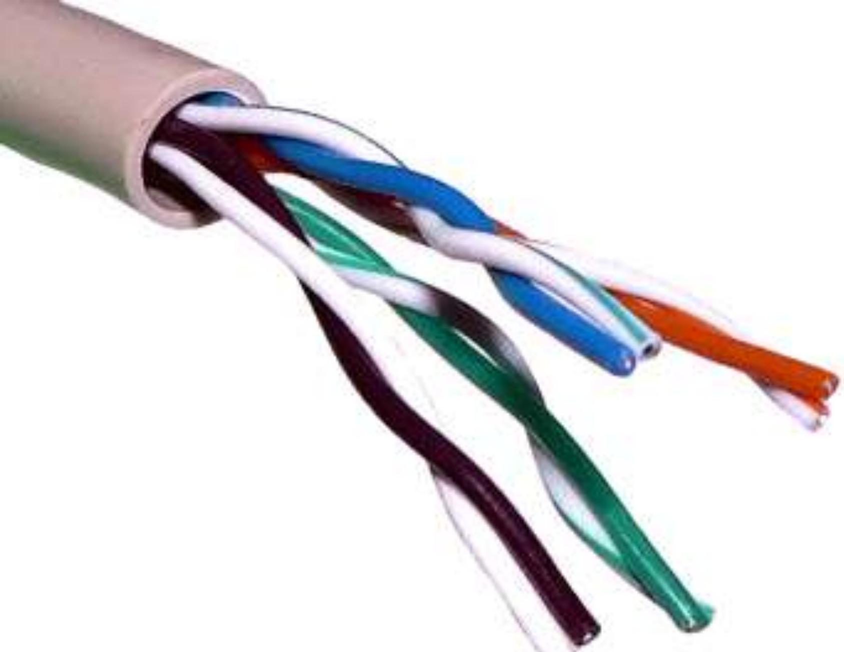


$$\gamma \equiv \sqrt{(R' + j\omega L')(G' + j\omega C')} = \alpha + j\beta$$

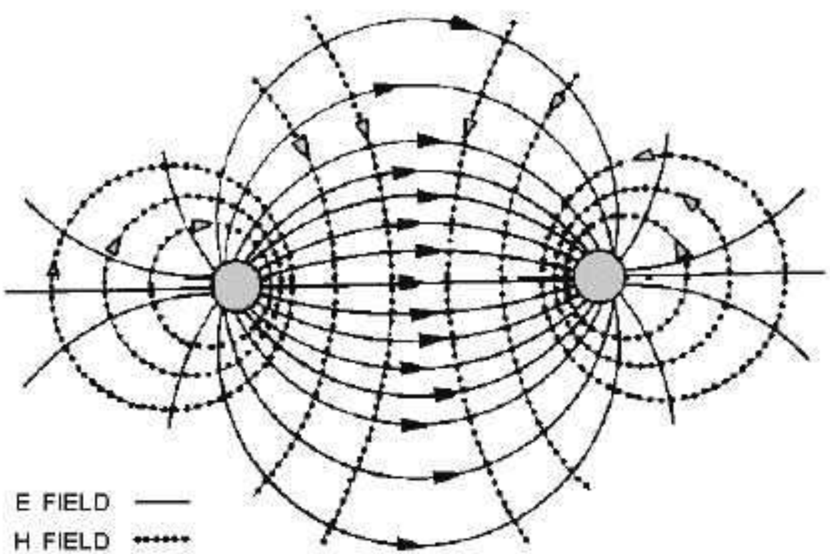
$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

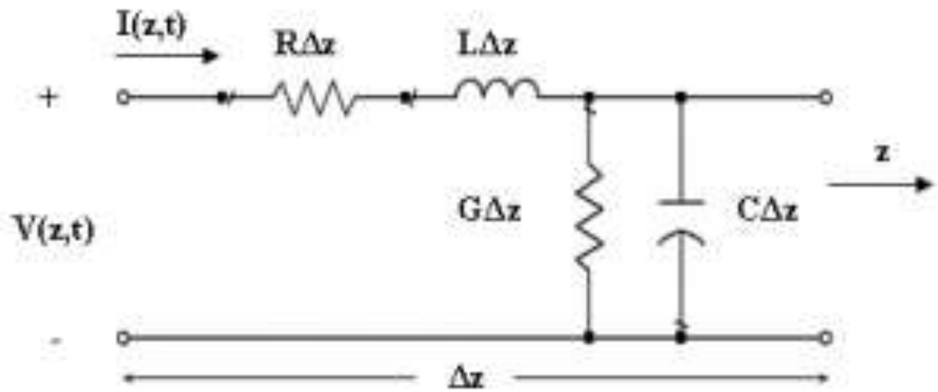
$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-}$$



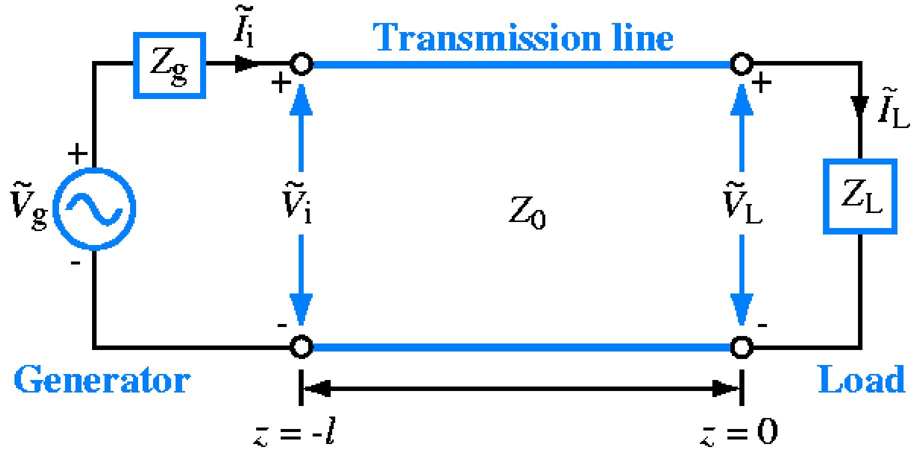
(a)





(b)

T-Lines (from Last Time)



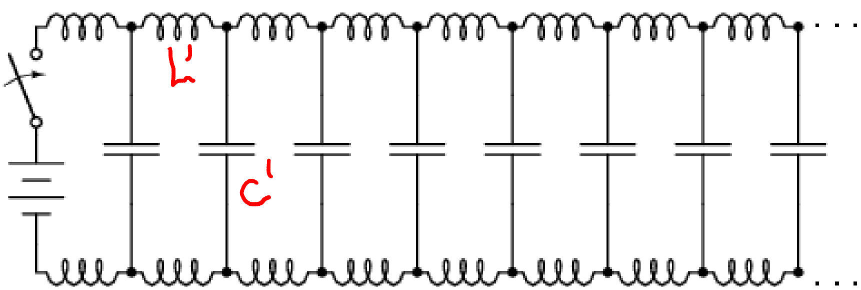
$$\gamma \equiv \sqrt{(R' + j\omega L')(G' + j\omega C')} = \alpha + j\beta$$

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

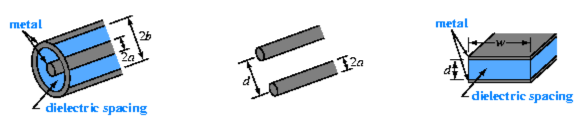
$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-}$$

Lossless T-Line (special case)



$R' = G' = 0$



(a) Coaxial line (b) Two-wire line (c) Parallel-plate line

(when is this a good approx.?)

$$\gamma = \sqrt{j\omega L' \cdot j\omega C'} = j\omega\sqrt{L'C'}$$

$\Rightarrow \alpha = 0$ = Lossless!

$\beta = \omega\sqrt{L'C'}$

Remember LC from 210? ☺

$$Z_0 = \sqrt{\frac{j\omega L'}{j\omega C'}} = \sqrt{\frac{L'}{C'}}$$ = purely Real

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\dfrac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless $(R' = G' = 0)$	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (60/\sqrt{\varepsilon_r})\ln(b/a)$
Lossless two wire	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120/\sqrt{\varepsilon_r}) \cdot \ln[(d/2a) + \sqrt{(d/2a)^2 - 1}]$ $Z_0 \simeq (120/\sqrt{\varepsilon_r})\ln(d/a)$, if $d \gg a$
Lossless parallel plate	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120\pi/\sqrt{\varepsilon_r})(d/w)$

Notes: (1) $\mu = \mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $c = 1/\sqrt{\mu_0\varepsilon_0}$, and $\sqrt{\mu_0/\varepsilon_0} \simeq (120\pi)\ \Omega$, where ε_r is the permittivity of insulating material. (2) For coaxial line, a and b are radii of inner and outer c (3) For two-wire line, a = wire radius and d = separation between wire centers. (4) For p line, w = width of plate and d = separation between the plates.

For Lossless T-Lines

u_p = phase velocity

$= \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{L'C'}}$

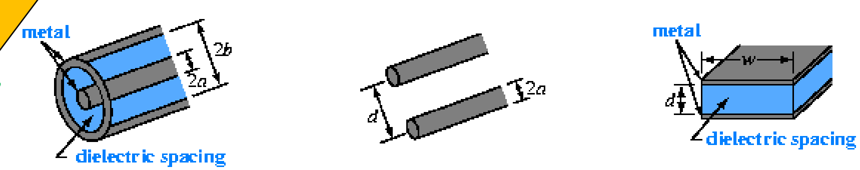
$= f\lambda$ with $\omega = 2\pi f$

Note: $\dfrac{1}{\sqrt{L'C'}}$ is not a function of frequency!

$\Rightarrow$ Lossless T-Lines are non-dispersive

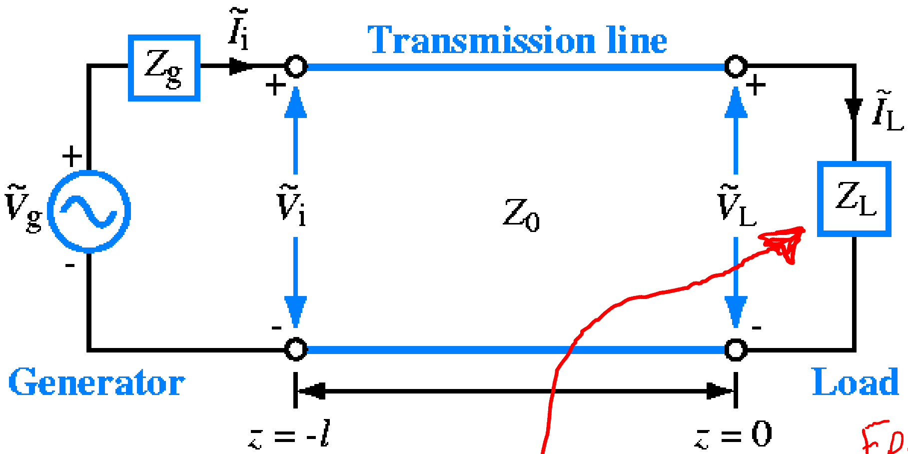
See HW for dispersionless line: R'C'=L'G'

Note: we will get to all the μ & ε stuff later in the semester... suffice it to say that L', C', etc. depend on what the T-Line is made of!



(a) Coaxial line (b) Two-wire line (c) Parallel-plate line

Where do those leftward propagating waves come from??



for LossLess

$$\tilde V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z} \quad (1)$$

$$\tilde I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

$$= \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z} \quad (2)$$

From Defn. Z_0

$$Z_L = \frac{\tilde V_L}{\tilde I_L} \quad (3)$$

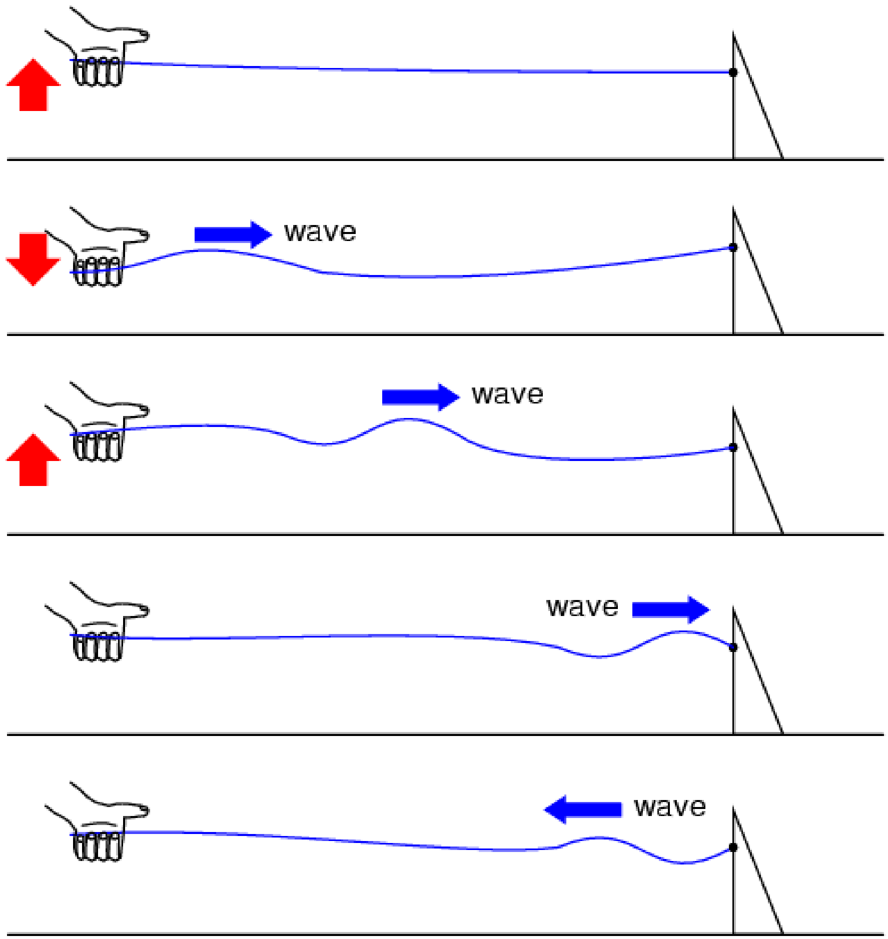
$$\frac{\text{Eqtn (1) at } z=0}{\text{Eqtn (2) at } z=0} = \frac{\tilde V_L}{\tilde I_L} = Z_L, \text{ from (3)}$$

$$\Rightarrow \quad V_0^- = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) V_0^+$$

Source wave amplitude

Reflected wave amplitude

Reflection Coefficient (Voltage), Γ



Reflection Coefficient

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1}$$

$$= |\Gamma| e^{j\theta_R}$$

in general, complex (even for lossless lines)

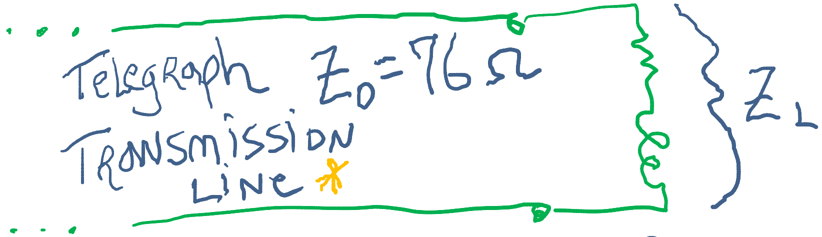
$$\frac{V_0^-}{V_0^+} = \Gamma \quad , \quad \frac{I_0^-}{I_0^+} = -\Gamma$$

Note: If $Z_L = Z_0$, $\Gamma = 0$

$\Rightarrow$ no reflection!

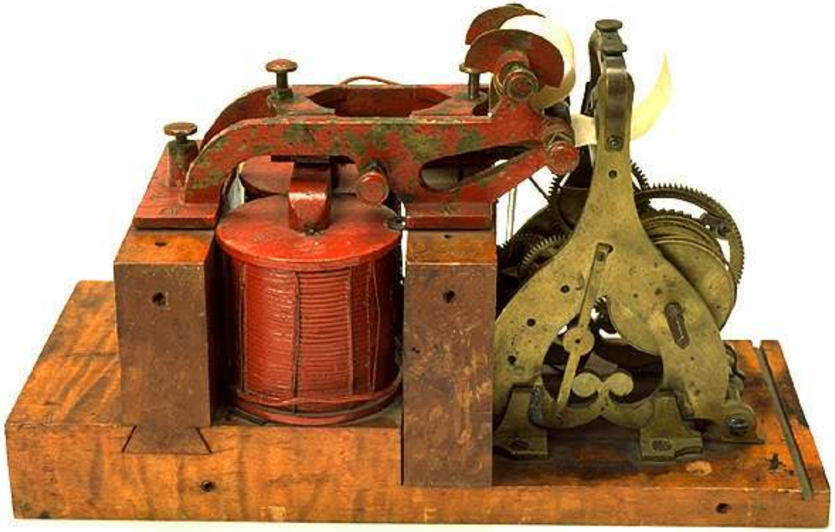
Matched Load

Example



$R = 2\,\Omega$

$L = 1$ mH



* Lossless!

What is the Reflection Coefficient at the Load?

(oh yeah, assume 1 kHz)

$$Z_L = 2 + j\left(2\pi \times 1\times10^{3} \times 1 \times 10^{-3}\right)$$
$$= 2 + j\,6.28\ \Omega$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.94 - j\,0.16 = 0.95\angle -9.5^\circ$$

Yikes! it's mostly Reflected!

For a purely Reactive Load, can show that $|\Gamma| = 1$.

What's this all mean??

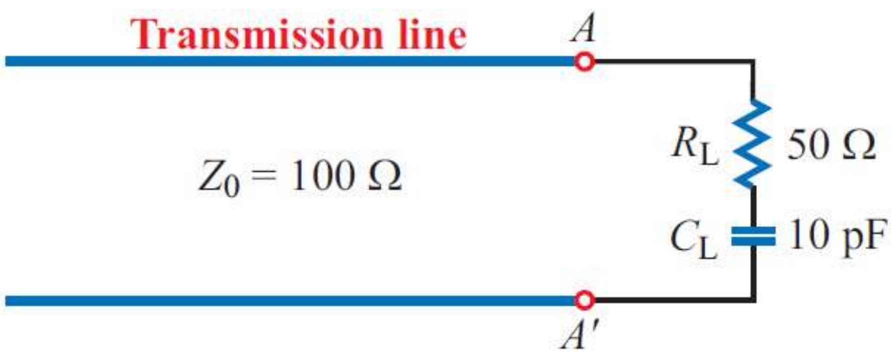
Example 2-3: Reflection Coefficient of a Series *RC* Load

A 100-Ω transmission line is connected to a load consisting of a 50-Ω resistor in series with a 10-pF capacitor. Find the reflection coefficient at the load for a 100-MHz signal.

Solution: The following quantities are given (Fig. 2-13):

$$R_L = 50\ \Omega, \qquad C_L = 10\ \text{pF} = 10^{-11}\ \text{F},$$

$$Z_0 = 100\ \Omega, \qquad f = 100\ \text{MHz} = 10^8\ \text{Hz}.$$



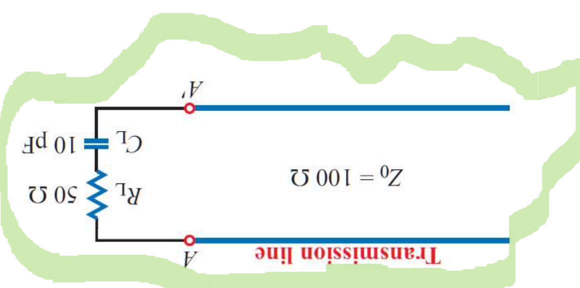
The normalized load impedance is

?

$$z_L = \frac{Z_L}{Z_0} = \frac{R_L - j/(\omega C_L)}{Z_0}$$

$$= \frac{1}{100}\left(50 - j\frac{1}{2\pi \times 10^8 \times 10^{-11}}\right)$$

$$= (0.5 - j1.59)\ \Omega.$$



$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

$$= \frac{0.5 - j1.59 - 1}{0.5 - j1.59 + 1}$$

$$= \frac{-0.5 - j1.59}{1.5 - j1.59} = \frac{-1.67e^{j72.6^\circ}}{2.19e^{-j46.7^\circ}} = -0.76e^{j119.3^\circ}.$$

This result may be converted into the form of Eq. (2.62) by replacing the minus sign with e^{-j180°. Thus,

$$\Gamma = 0.76e^{j119.3^\circ}e^{-j180^\circ} = 0.76e^{-j60.7^\circ} = 0.76\angle{-60.7^\circ},$$

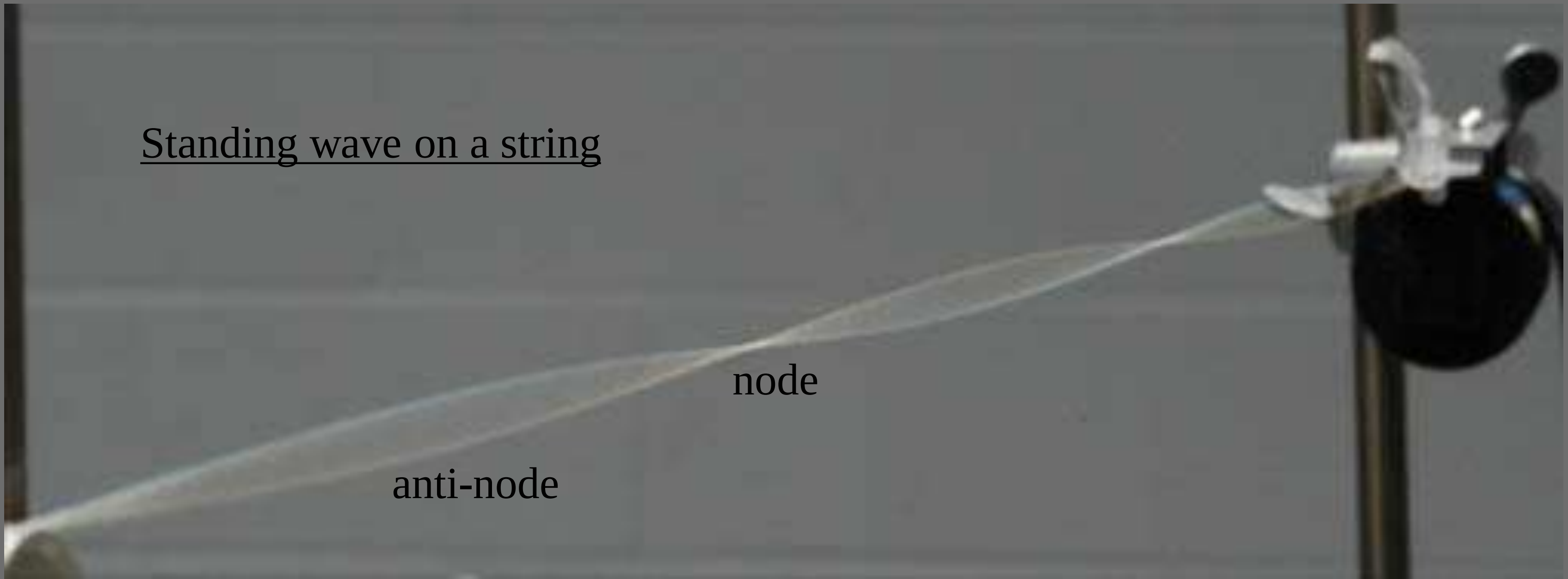
or

$$\boxed{|\Gamma| = 0.76, \qquad \theta_r = -60.7^\circ.}$$

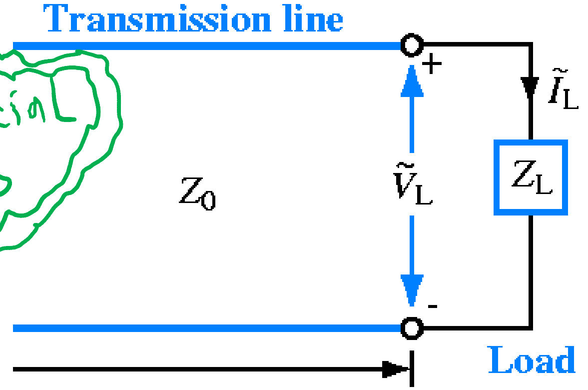
Transmitted wave and reflected wave *add* by superposition: $\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$

In this animation:
Green wave moves to the right (+z)
Blue wave (the reflected one, although we do not see the load in this animation) moves to the left (-z)
Yellow waveform is the superposition of both (and therefore the **total** voltage on the line). It is a <u>standing</u> wave! ☺

***Standing** Wave* animation (oscilloscope)
http://www.youtube.com/watch?v=ic73oZoqr70

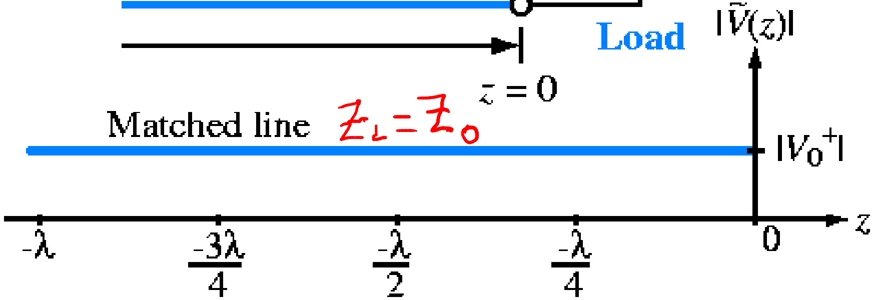


3 special cases

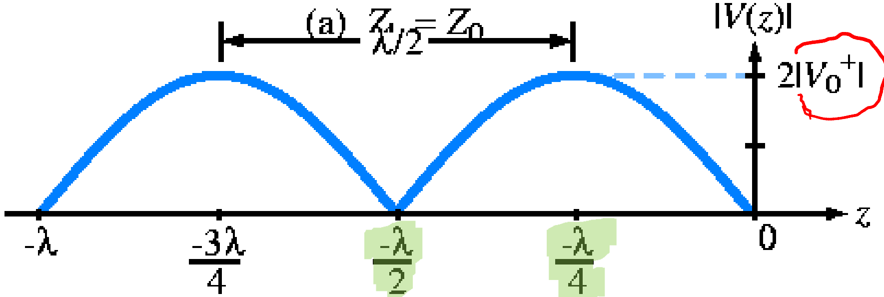


$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

The load represents a **boundary condition**!



(a) $Z_L = Z_0$

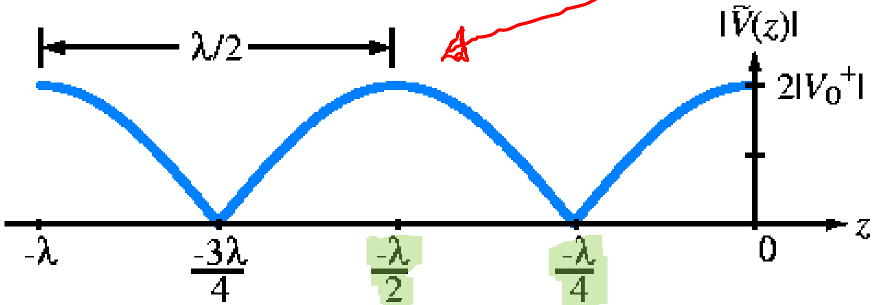


Amplitude of wave

$\Gamma = -1$

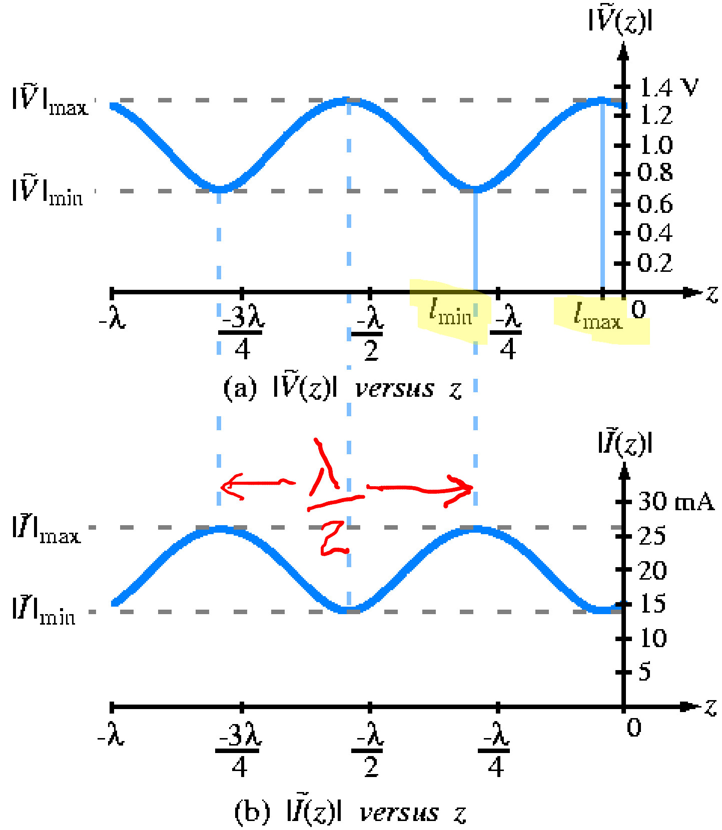
(b) $Z_L = 0$ (short circuit)

Point of max voltage is point of min current (and vice versa)



$\Gamma = 1$

(c) $Z_L = \infty$ (open circuit)



(a) $|\tilde V(z)|$ *versus* z

(b) $|\tilde I(z)|$ *versus* z

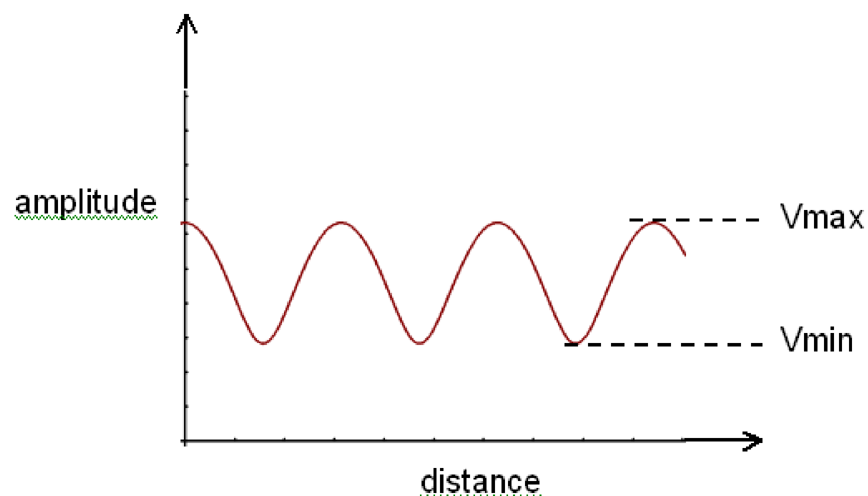
Note

- For a general Load, the max or min voltage is not necessarily at location $z = 0$ (the Load)
- For a general Load, there don't need to be true nodes (e.g. $|\tilde V| = 0$) anywhere.

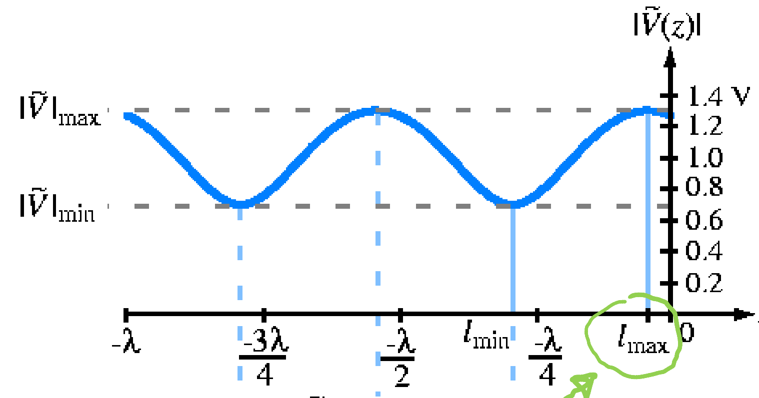
(Voltage) Standing Wave Ratio*

$$S \equiv \frac{|\tilde V|_{max}}{|\tilde V|_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

* VSWR or SWR



Exercise 2.10 A 140-Ω lossless line is terminated in a load impedance $Z_L = (280 + j182)\ \Omega$. If $\lambda = 72$ cm, find (a) the reflection coefficient Γ, (b) the voltage standing-wave ratio S, (c) the locations of voltage maxima, and (d) the locations of voltage minima.



(a) $|\tilde{V}(z)|$ versus z

Solution:

$$Z_0 = 140\ \Omega, \qquad Z_L = (280 + j182)\ \Omega$$

(a)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{280 + j182 - 140}{280 + j182 + 140} = \frac{140 + j182}{420 + j182} = 0.5\angle 29^\circ .$$

$= |\Gamma| \angle \theta_R$

the 1st max in the standing wave is located $l_{max} = \frac{\theta_R \lambda}{4\pi}$ from the load. (for Lossless Lines)

(b)

Note: if $|\Gamma| = 0$, then $S = 1$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.5}{1 - 0.5} = \frac{1.5}{0.5} = 3.$$

(c)

what does $|\Gamma| = 0$ correspond to?

$$l_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2}, \qquad n = 0, 1, 2, \ldots$$
$$= \frac{(29\pi/180) \times 0.72}{4\pi} + \frac{n \times 0.72}{2}$$
$$= (2.9 + 36n) \quad \text{(cm)}, \qquad n = 0, 1, 2, \ldots$$

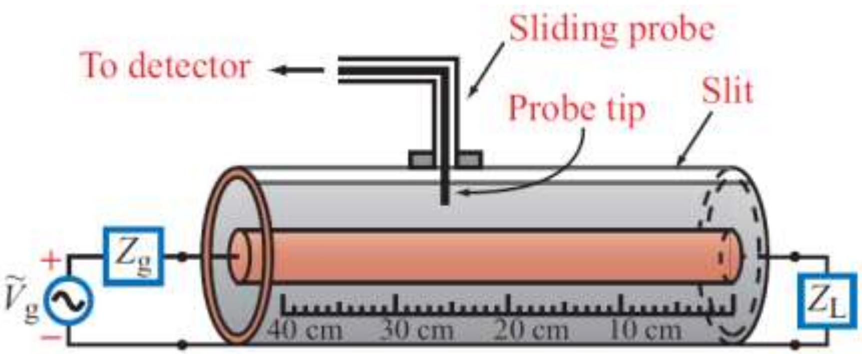
(d)

$$l_{min} = l_{max} + \frac{\lambda}{4}$$
$$= \left[(2.9 + 36n) + \frac{72}{4}\right] \text{ cm}$$
$$= (20.9 + 36n) \text{ cm}, \qquad n = 0, 1, 2, \ldots$$

Q: what if $|\Gamma| = 1$? what's that mean? what is SWR?

WTF?

Example 2-6: Measuring Z_L with a Slotted Line



Solution: The following quantities are given:

$$Z_0 = 50\ \Omega,$$
$$S = 3,$$
$$d_{min} = 12\text{ cm}.$$

Since the distance between successive voltage minima is $\lambda/2$,

$$\lambda = 2 \times 0.3 = 0.6\text{ m},$$

and

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{0.6} = \frac{10\pi}{3} \qquad \text{(rad/m)}.$$

From Eq. (2.73), solving for $|\Gamma|$ in terms of S gives

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = 0.5.$$

Next, we use the condition given by Eq. (2.71) to find θ_r:

$$2\beta d_{min} - \theta_r = \pi, \qquad \text{for } n = 0 \text{ (first minimum)},$$

which gives

$$\theta_r = 2\beta d_{min} - \pi = 2 \times \frac{10\pi}{3} \times 0.12 - \pi = -0.2\pi \text{ (rad)} = -36°.$$

Hence,

$$\Gamma = |\Gamma| e^{j\theta_r} = 0.5 e^{-j36°} = 0.405 - j0.294.$$

Solving Eq. (2.59) for Z_L, we have

$$Z_L = Z_0 \left[\frac{1+\Gamma}{1-\Gamma}\right] = 50 \left[\frac{1 + 0.405 - j0.294}{1 - 0.405 + j0.294}\right] \boxed{= (85 - j67)\ \Omega.}$$

EXAMPLE (S)

CIRCLE (T)RUE or (F)ALSE[2]

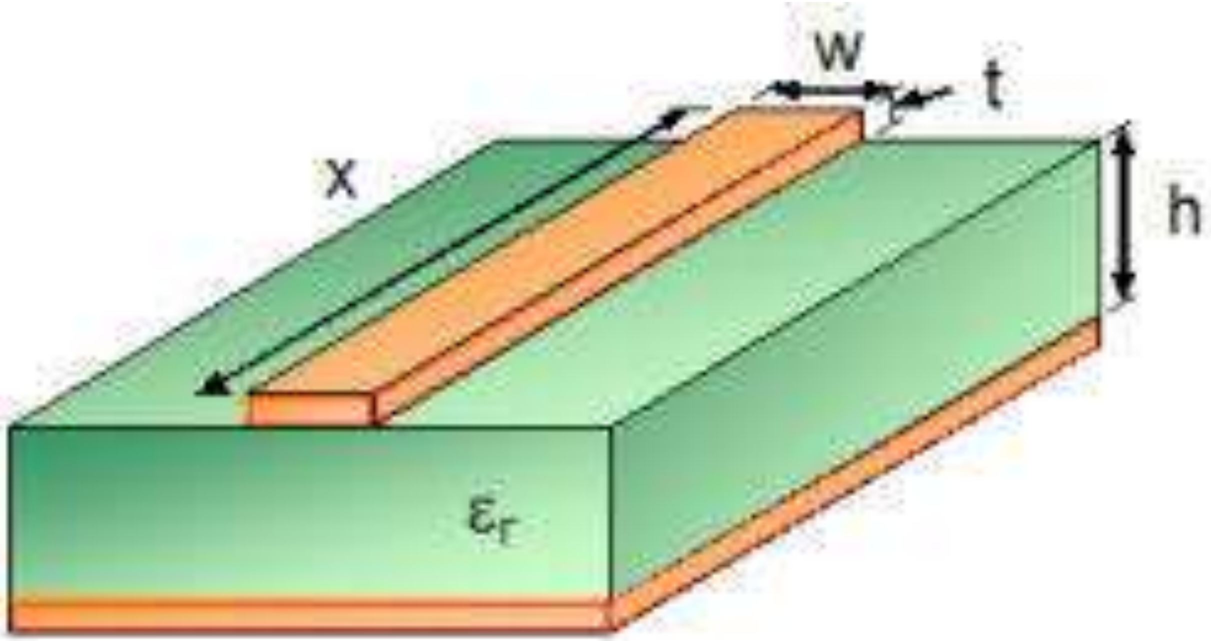
5) Z_o is only defined for lossless transmission lines. T F

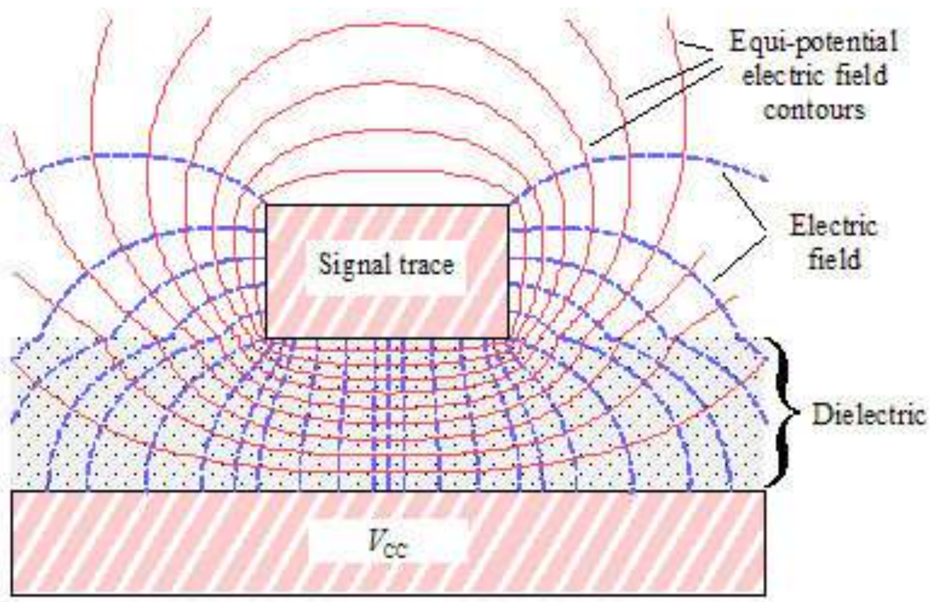
6) You do **not** have to treat a 6 foot long power cord as a transmission line. T F

3) Given a lossless transmission line of known intrinsic impedance, phase speed and length, can you determine the load impedance from measurements of the input impedance? Explain.

http://www.amanogawa.com/archive/transmissionA.html

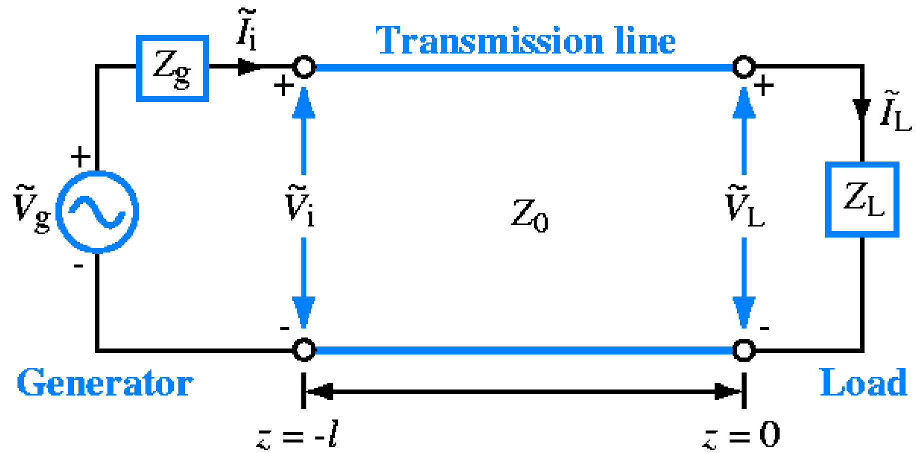
Lecture 6





http://www.naic.edu/reu_program.html
http://www.haystack.mit.edu/edu/reu/
http://www.csnr.usra.edu/
http://www.unl.edu/summerprogram/
http://www.vs.afrl.af.mil/SpaceScholars/

PLUS lots more !!!! ☺



Reflection

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{-I_0^-}{I_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Note: function of frequency!

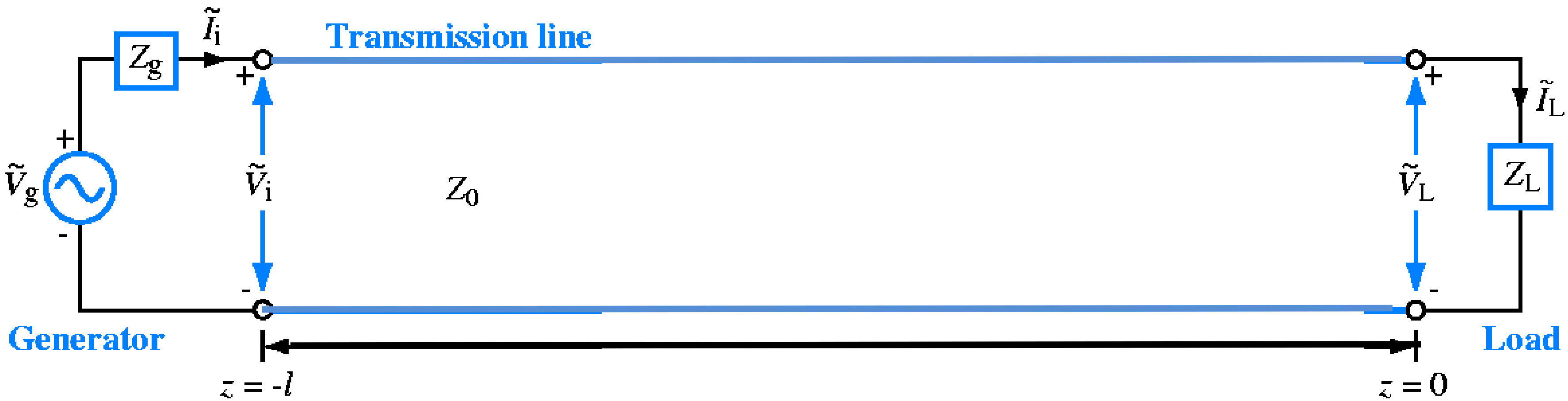
Standing Waves

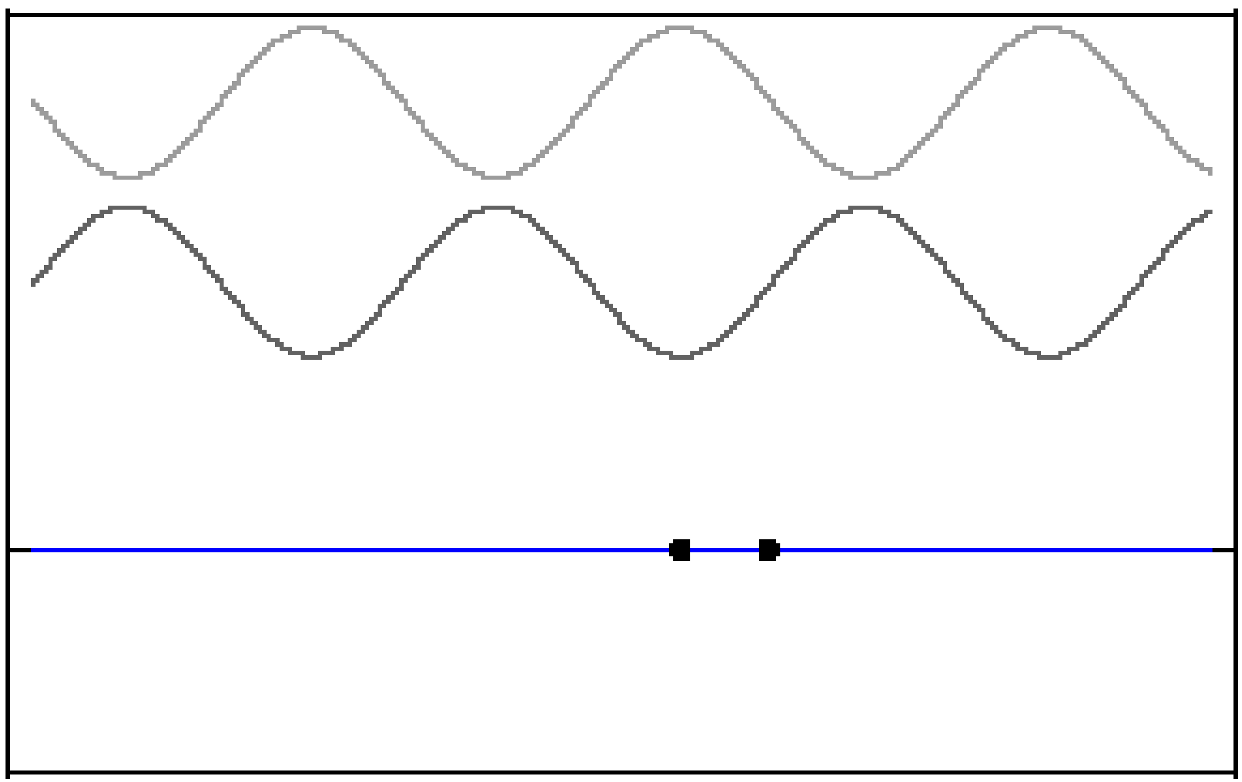
***Standing** Wave* animation (oscilloscope)
http://www.youtube.com/watch?v=ic73oZoqr70

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

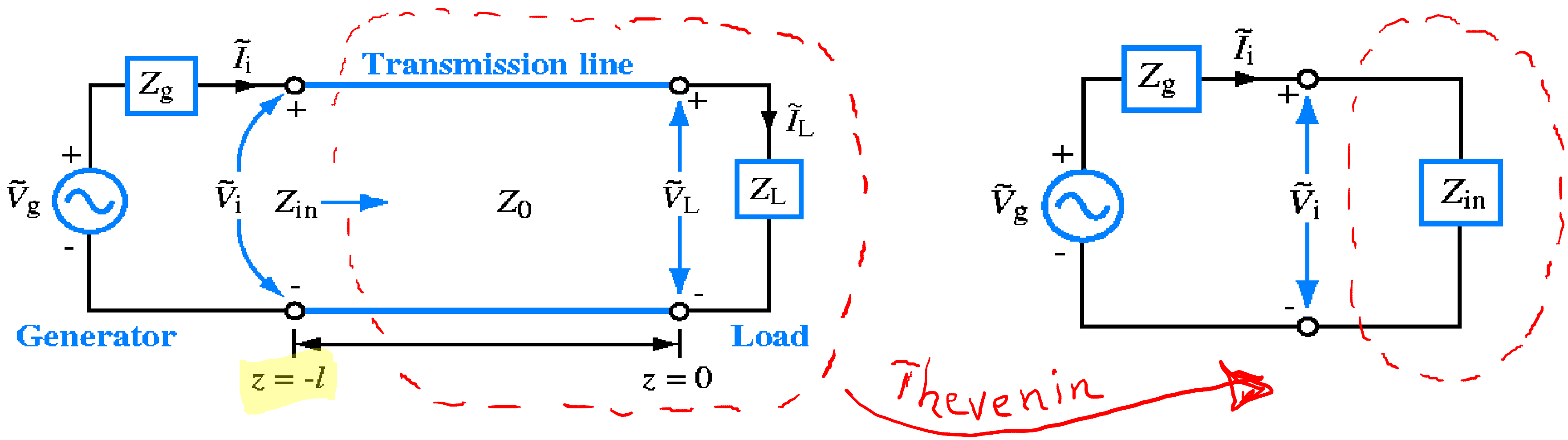
Note: $\tilde{V}$ & $\tilde{I}$ are functions of position z

(see slotted Line ☺)





What is the load Z_L in this case?



Input Impedance

Lossless

$$Z_{in} = \frac{\tilde{V}(-\ell)}{\tilde{I}(-\ell)} = \frac{V_0^+ e^{-j\beta(-\ell)} + \overbrace{V_0^-}^{\Gamma V_0^+} e^{+j\beta(-\ell)}}{\underbrace{I_0^+}_{V_0^+/Z_0} e^{-j\beta(-\ell)} + \underbrace{I_0^-}_{-\Gamma I_0^+} e^{+j\beta(-\ell)}} = Z_0 \left[\frac{1+\Gamma e^{-j2\beta\ell}}{1-\Gamma e^{-j2\beta\ell}}\right]$$

$$\beta = \mathrm{Im}\{\gamma\} = \frac{2\pi}{\lambda}$$

Notes:

* Z_{in} is periodic in ℓ — so can write in terms of trig, too
* $Z_{in} \neq Z_0$
* $\tilde{V}_i = \left(\frac{Z_{in}}{Z_{in}+Z_g}\right)\tilde{V}_g = V_0^+(-\ell) + V_0^-(-\ell)$

or, equivalently:

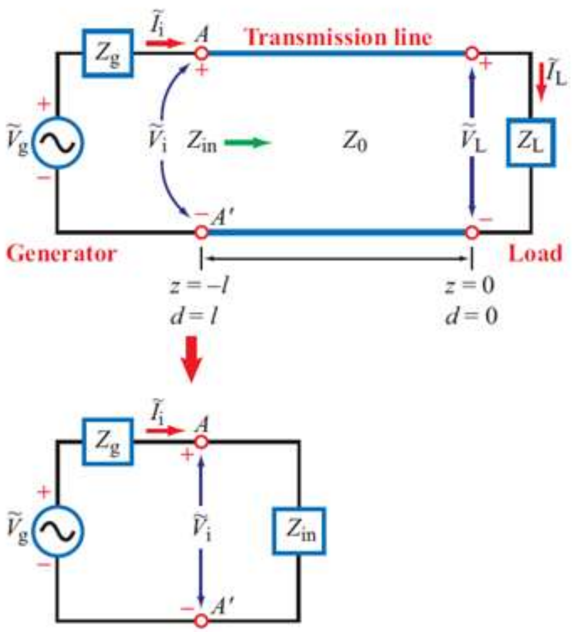
$$\boxed{Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}\right]}$$

Example 2-7: Complete Solution for $v(z,t)$ and $i(z,t)$

A 1.05-GHz generator circuit with series impedance $Z_g = 10\ \Omega$ and voltage source given by

$$v_g(t) = 10\sin(\omega t + 30^\circ) \qquad \text{(V)}$$

is connected to a load $Z_L = (100 + j50)\ \Omega$ through a 50-Ω, 67-cm long lossless transmission line. The phase velocity of



the line is 0.7c, where c is the velocity of light in a vacuum. Find $v(z,t)$ and $i(z,t)$ on the line.

Solution: From the relationship $u_p = \lambda f$, we find the wavelength

$$\lambda = \frac{u_p}{f} = \frac{0.7 \times 3 \times 10^8}{1.05 \times 10^9} = 0.2\ \text{m},$$

and

$$\beta l = \frac{2\pi}{\lambda} l = \frac{2\pi}{0.2} \times 0.67 = 6.7\pi = 0.7\pi = 126^\circ,$$

where we have subtracted multiples of 2π. The voltage reflection coefficient at the load is

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(100 + j50) - 50}{(100 + j50) + 50} = 0.45e^{j26.6^\circ}.$$

$$Z_{in} = Z_0\left(\frac{1+\Gamma_l}{1-\Gamma_l}\right) = Z_0\left(\frac{1+\Gamma e^{-j2\beta l}}{1-\Gamma e^{-j2\beta l}}\right) = 50\left(\frac{1+0.45e^{j26.6^\circ}e^{-j252^\circ}}{1-0.45e^{j26.6^\circ}e^{-j252^\circ}}\right) = (21.9 + j17.4)\ \Omega.$$

Rewriting the expression for the generator voltage with the cosine reference, we have

$$\begin{aligned} v_g(t) &= 10\sin(\omega t + 30^\circ) \\ &= 10\cos(90^\circ - \omega t - 30^\circ) \\ &= 10\cos(\omega t - 60^\circ) \\ &= \Re\mathfrak{e}[10e^{-j60^\circ}e^{j\omega t}] = \Re\mathfrak{e}[\widetilde{V}_g e^{j\omega t}] \qquad \text{(V)}. \end{aligned}$$

Hence, the phasor voltage $\widetilde{V}_g$ is given by

$$\widetilde{V}_g = 10\,e^{-j60^\circ} = 10\angle{-60^\circ} \qquad \text{(V)}.$$

Application of Eq. (2.82) gives

$$\begin{aligned} V_0^+ &= \left(\frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right) \\ &= \left[\frac{10e^{-j60^\circ}(21.9 + j17.4)}{10 + 21.9 + j17.4}\right] \\ &\quad \cdot (e^{j126^\circ} + 0.45e^{j26.6^\circ}e^{-j126^\circ})^{-1} \\ &= 10.2e^{j159^\circ} \qquad \text{(V)}. \end{aligned}$$

Using Eq. (2.63a) with $z = -d$, the phasor voltage on the line is

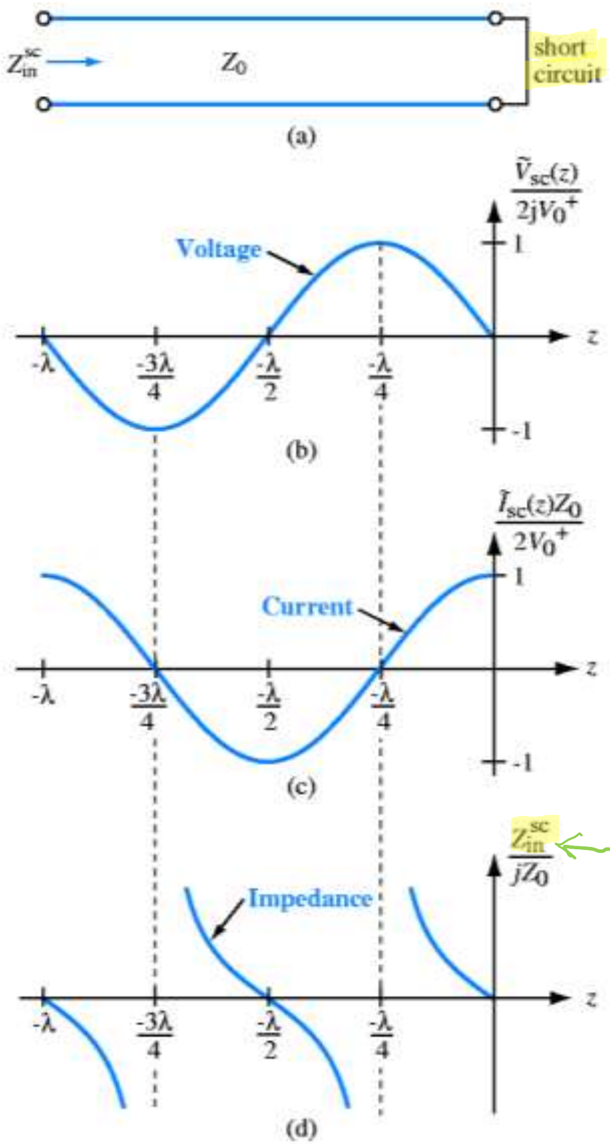
$$\begin{aligned} \widetilde{V}(d) &= V_0^+(e^{j\beta d} + \Gamma e^{-j\beta d}) \\ &= 10.2e^{j159^\circ}(e^{j\beta d} + 0.45e^{j26.6^\circ}e^{-j\beta d}), \end{aligned}$$

and the corresponding instantaneous voltage $v(d,t)$ is

$$\boxed{\begin{aligned} v(d,t) &= \Re\mathfrak{e}[\widetilde{V}(d)\,e^{j\omega t}] \\ &= 10.2\cos(\omega t + \beta d + 159^\circ) \\ &\quad + 4.55\cos(\omega t - \beta d + 185.6^\circ) \qquad \text{(V)}. \end{aligned}}$$

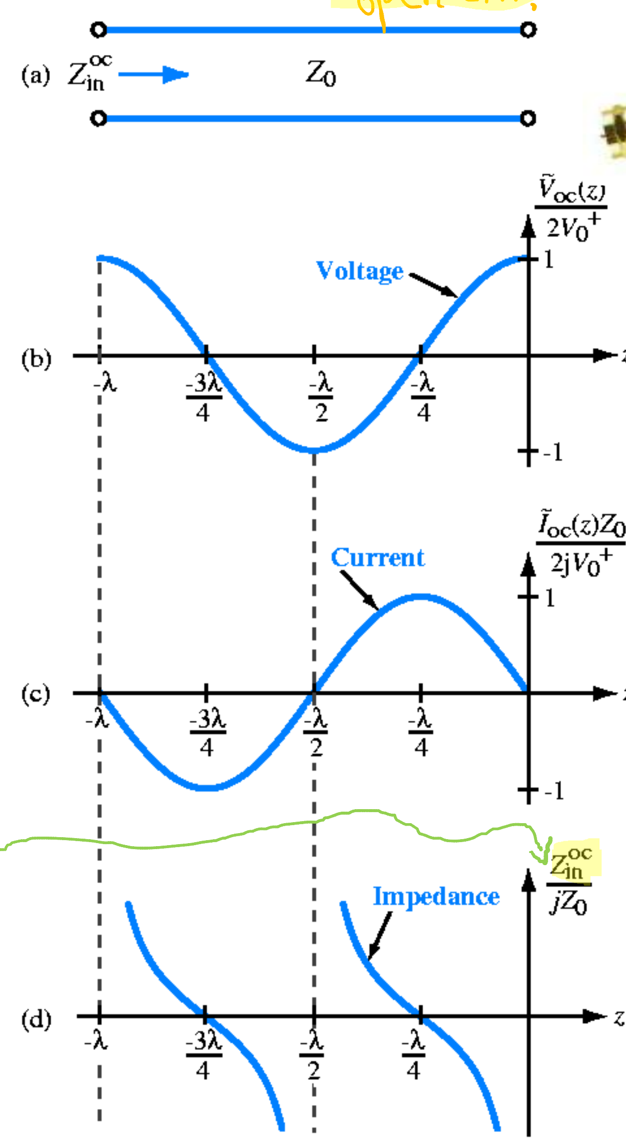
Similarly, Eq. (2.63b) leads to

$$\boxed{\begin{aligned} \widetilde{I}(d) &= 0.20e^{j159^\circ}(e^{j\beta d} - 0.45e^{j26.6^\circ}e^{-j\beta d}), \\ i(d,t) &= 0.20\cos(\omega t + \beta d + 159^\circ) \\ &\quad + 0.091\cos(\omega t - \beta d + 185.6^\circ) \qquad \text{(A)}. \end{aligned}}$$



can get as short or as 50 Ω

open ckt.



which way is "LEFT"?

Purely Reactive

Special Cases

$\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

Voltage maximum Voltage minimum	$\lvert\widetilde{V}\rvert_{max} = \lvert V_0^+\rvert[1+\lvert\Gamma\rvert]$ $\lvert\widetilde{V}\rvert_{min} = \lvert V_0^+\rvert[1-\lvert\Gamma\rvert]$
Positions of voltage maxima (also positions of current minima) Position of first maximum (also position of first current minimum)	$l_{max} = \dfrac{\theta_r\lambda}{4\pi} + \dfrac{n\lambda}{2}, \quad n = 0,1,2,\dots$ $l_{max} = \begin{cases} \dfrac{\theta_r\lambda}{4\pi}, & \text{if } 0 \le \theta_r \le \pi \\ \dfrac{\theta_r\lambda}{4\pi} + \dfrac{\lambda}{2}, & \text{if } -\pi \le \theta_r \le 0 \end{cases}$
Positions of voltage minima (also positions of first current maxima) Position of first minimum (also position of first current maximum)	$l_{min} = \dfrac{\theta_r\lambda}{4\pi} + \dfrac{(2n+1)\lambda}{4}, \quad n = 0,1,2,\dots$ $l_{min} = \dfrac{\lambda}{4}\left(1 + \dfrac{\theta_r}{\pi}\right)$
Input impedance	$Z_{in} = Z_0\left(\dfrac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right) = Z_0\left[\dfrac{1+\Gamma e^{-j2\beta l}}{1-\Gamma e^{-j2\beta l}}\right]$
Positions at which Z_{in} is real	at voltage maxima and minima
Z_{in} at voltage maxima	$Z_{in} = Z_0\left(\dfrac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}\right)$
Z_{in} at voltage minima	$Z_{in} = Z_0\left(\dfrac{1-\lvert\Gamma\rvert}{1+\lvert\Gamma\rvert}\right)$
Z_{in} of short-circuited line	$Z_{in}^{sc} = jZ_0\tan\beta l$
Z_{in} of open-circuited line	$Z_{in}^{oc} = -jZ_0\cot\beta l$
Z_{in} of line of length $l = n\lambda/2$	$Z_{in} = Z_L, \quad n = 0,1,2,\dots$
Z_{in} of line of length $l = \lambda/4 + n\lambda/2$	$Z_{in} = Z_0^2/Z_L, \quad n = 0,1,2,\dots$
Z_{in} of matched line	$Z_{in} = Z_0 = Z_L$

$\lvert V_0^+\rvert$ = amplitude of incident wave, $\Gamma = \lvert\Gamma\rvert e^{j\theta_r}$ with $-\pi < \theta_r < \pi$; θ_r in radians.

Always check out Limiting cases, like $l=0$

Measuring β

$\tan\beta l = \sqrt{\dfrac{-Z_{in}^{sc}}{Z_{in}^{oc}}}$

Beware phase wrap around Ambiguity!

Frequency Dependance Matters!

Measuring Z_0

$Z_0 = \sqrt{Z_{in}^{sc} Z_{in}^{oc}}$

for lossless, not function of freq.!

 =  or  ??

Example:

Exercise 2.11 A 50-Ω lossless transmission line uses an insulating material with $\varepsilon_r = 2.25$. When terminated in an open circuit, how long should the line be for its input impedance to be equivalent to a 10-pF capacitor at 50 MHz?

$Z_L \to \infty$

$\Gamma = 1$

Solution: For a 10-pF capacitor at 50 MHz,

Desired Equivalent Capacitance

$$Z_c = \frac{1}{j\omega C} = \frac{-j}{2\pi \times 50 \times 10^6 \times 10 \times 10^{-12}} = -j\frac{1000}{\pi}\ \Omega$$

$\operatorname{Im}\{\gamma\}$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi\sqrt{\varepsilon_r}}{\lambda_0} = \frac{2\pi f\sqrt{\varepsilon_r}}{c}$$

$$= \frac{2\pi \times 5 \times 10^7 \sqrt{2.25}}{3 \times 10^8} = 1.57 \quad \text{(rad/m)}.$$

$\lambda = 4\,\text{m}$

For lossless lines with open-circuit termination,

$$Z_{in} = -jZ_0 \cot \beta l = -j50 \cot 1.57l = Z_0\left[\frac{1+\Gamma e^{-j2\beta l}}{1-\Gamma e^{-j2\beta l}}\right]$$

Works for any lossless Line!

Hence,

$$-j\frac{1000}{\pi} = -j50 \cot 1.57l$$

or

$$l = 10 \quad \text{(cm)}.$$

$\pm \sim 1\%$ of λ (a question of precision)

Q: Would you use an open or closed ckt in practice?

Lines of Length $\ell = n\lambda/2$ and $\ell = n\frac{\lambda}{4}$

* $\ell = \frac{n\lambda}{2}$ for $n = 0, 1, 2, \ldots \Rightarrow Z_{in} = Z_L$, i.e. as if T-Line isn't there!

  HALF-WAVE LINE

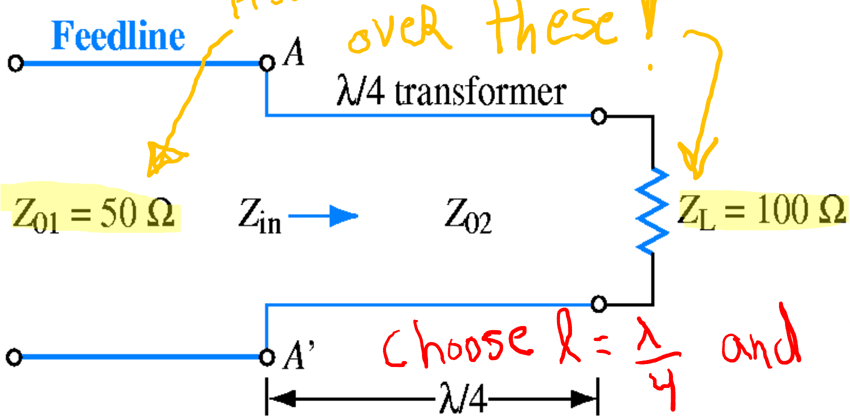
NOTE: For Lossless and single Freq.

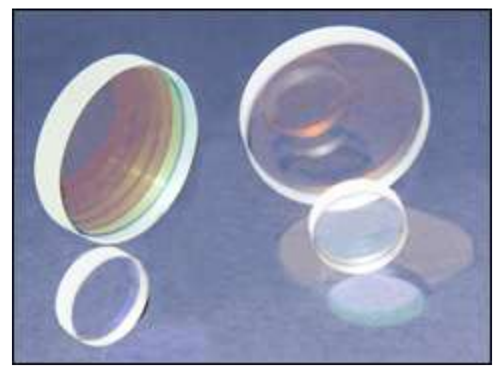
* $\ell = \frac{n\lambda}{4}$ for $n = 1, 3, 5, \ldots \Rightarrow Z_{in} = Z_0^2 / Z_L$

  QUARTER WAVE LINE

Example

Assume no control over these!



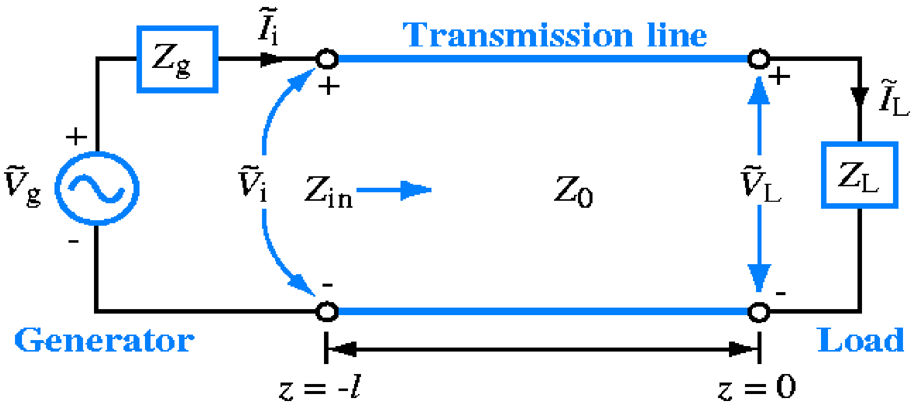


Choose $\ell = \frac{\lambda}{4}$ and $Z_{0_2} = \sqrt{100\,\Omega \times 50\,\Omega} = 70.7\,\Omega$

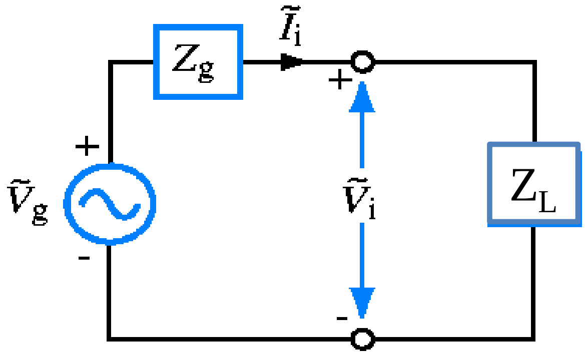
Why?

If length is *SHORT*, what is Z_{in}??

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right]$$



$$\beta l << 1$$



It's a T-line if:

$\frac{l}{\lambda} \sim \frac{fl}{c} \gtrsim 1\% \Rightarrow$ 3 MHz ~ m's
3 GHz ~ mm's

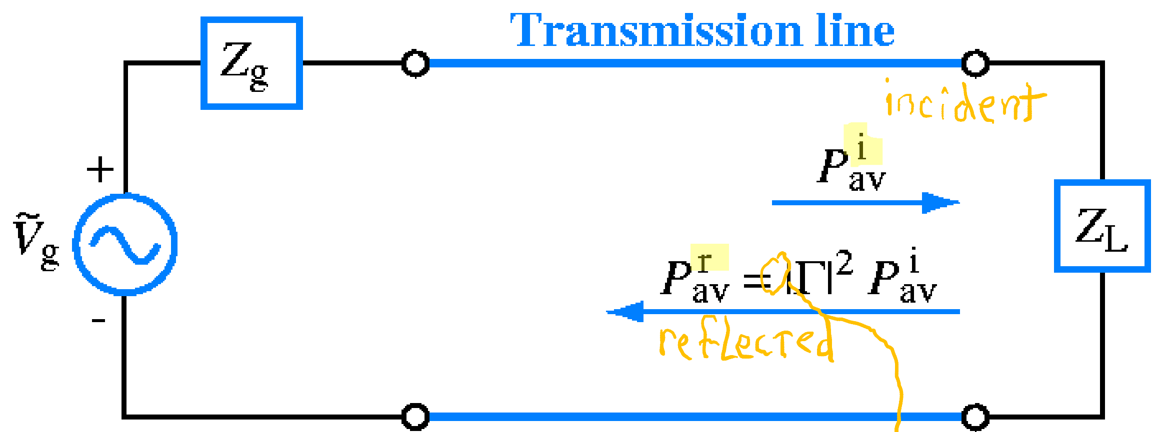
Power

Instantaneous Power

$\Leftrightarrow \quad v_i(t)\, i_i(t)$

$\underbrace{}_{\text{Time Domain}}$

$$\underbrace{= \frac{|V_0^+|^2}{Z_0} \cos^2(\omega t + \phi^+)}_{\text{using phasor \#'s, but still in time domain!}}$$



incident

reflected

depends on how you define direction...

Time-Average Power

$$P^i_{av} = \frac{|V_0^+|^2}{2 Z_0} \text{ W}, \quad P^r_{av} = -|\Gamma|^2 P^i_{av}$$

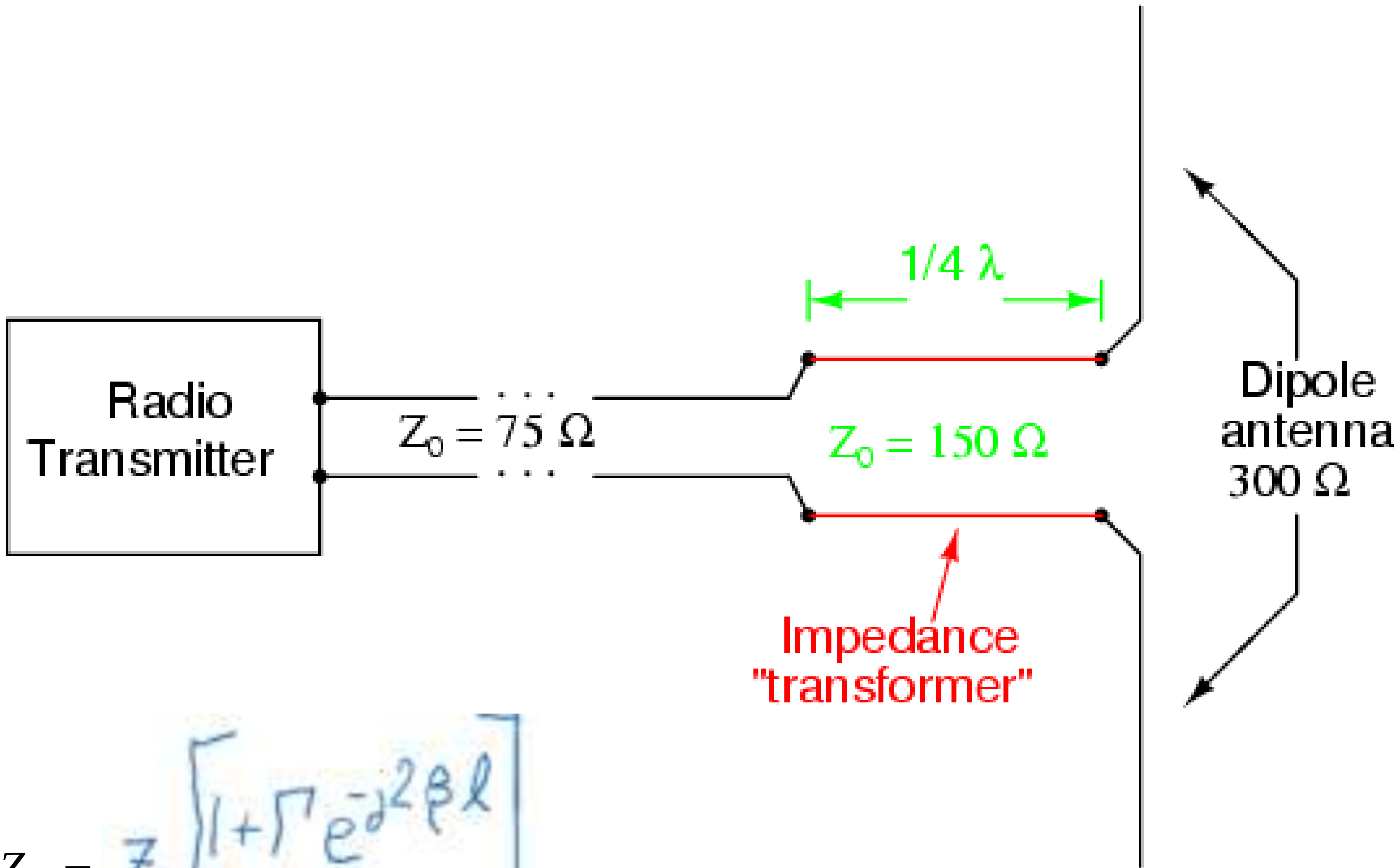
$$= \underbrace{\frac{1}{2} \operatorname{Re}\left[\tilde{V} \cdot \tilde{I}^*\right]}_{\text{phasor domain}}$$

Note: Power delivered to Load is $P^i_{AV} - P^r_{AV}$

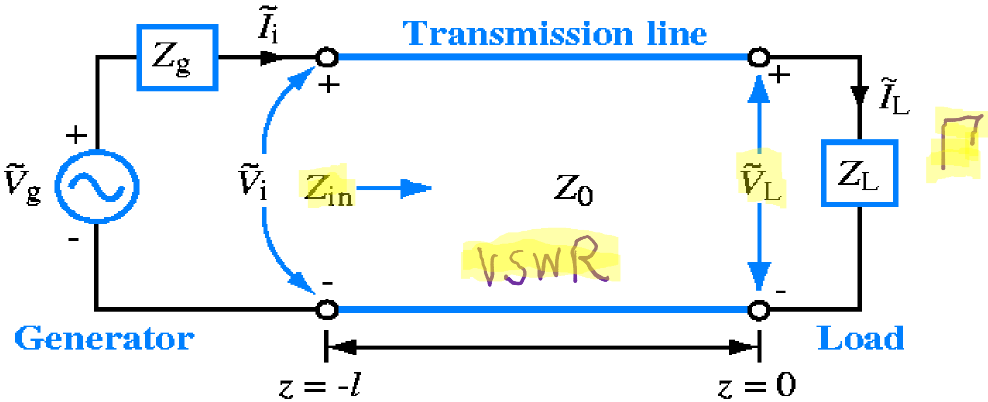
$Z_0 = 75\,\Omega$

$Z_0 = 300\,\Omega$

Lecture 7



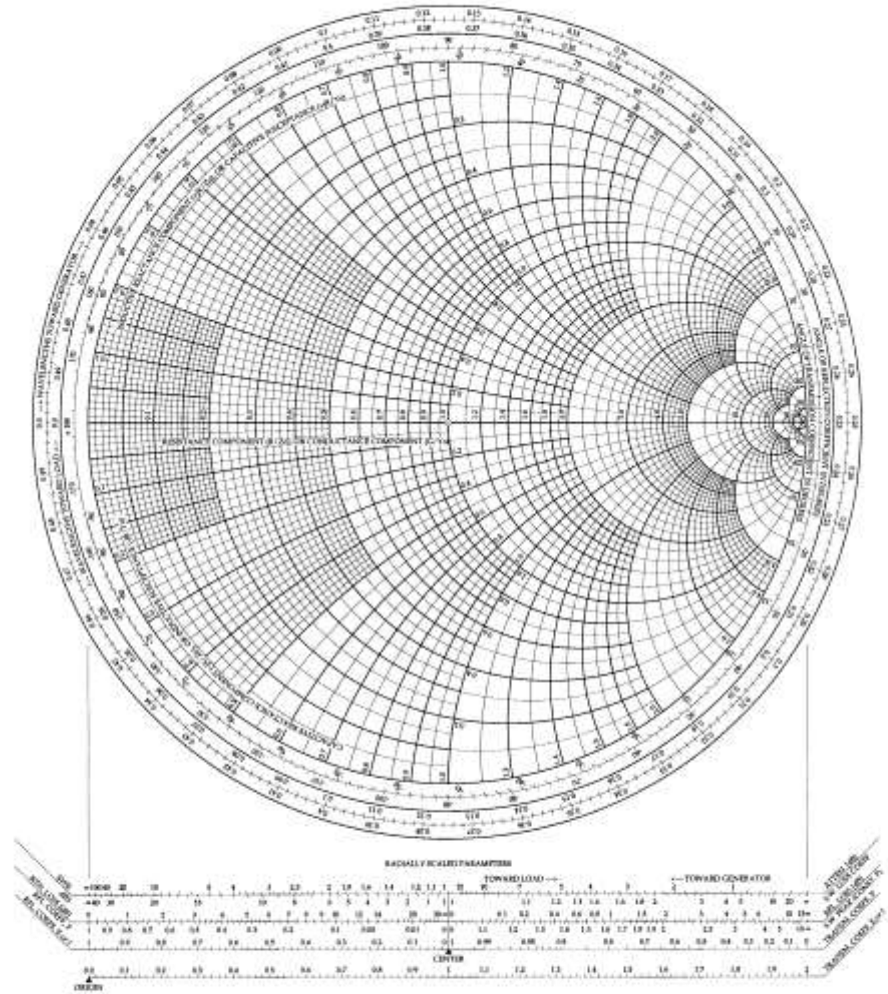
$$Z_{in} = Z_0 \left[\frac{1 + \Gamma e^{-j2\beta \ell}}{1 - \Gamma e^{-j2\beta \ell}} \right] \quad or \quad Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right]$$



Why?

The Complete Smith Chart

Black Magic Design



Smith's Chart

What is Smith's chart? A graphical tool.

Purpose of introducing Smith's chart: To avoid heavy math and speed up the process of analyzing and designing transmission line circuits.

How is it constructed? On the basis of the reflection coefficient plane.

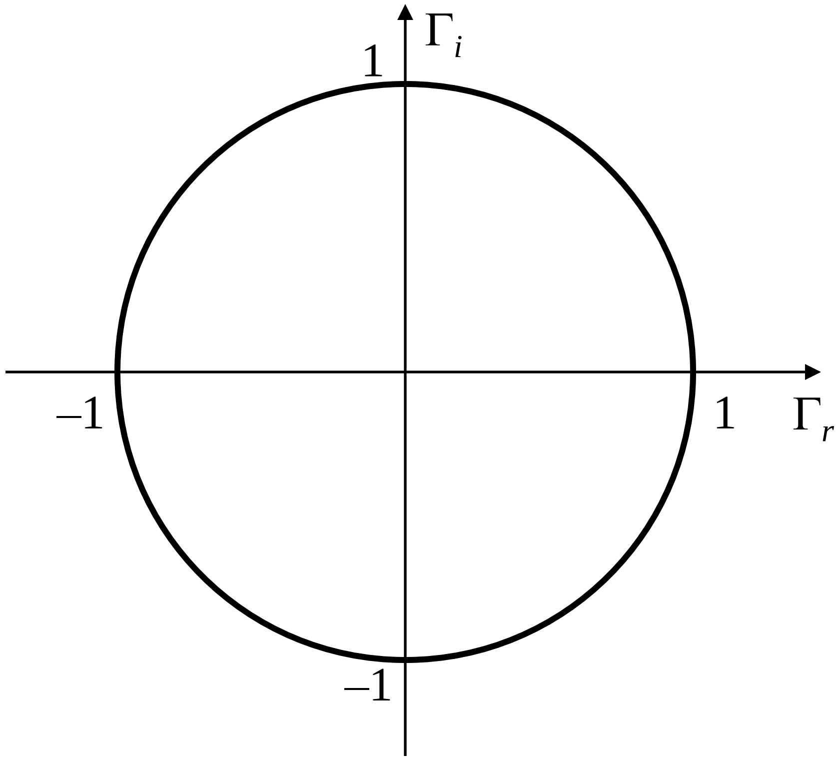
$$\Gamma = |\Gamma| e^{j\theta_\Gamma} = \Gamma_r + j\Gamma_i \;, \quad \text{where } \Gamma_r = \mathrm{Re}\{\Gamma\} \text{ and } \Gamma_i = \mathrm{Im}\{\Gamma\}$$

Only a circle of radius 1 has a meaning since $|\Gamma| \leq 1$

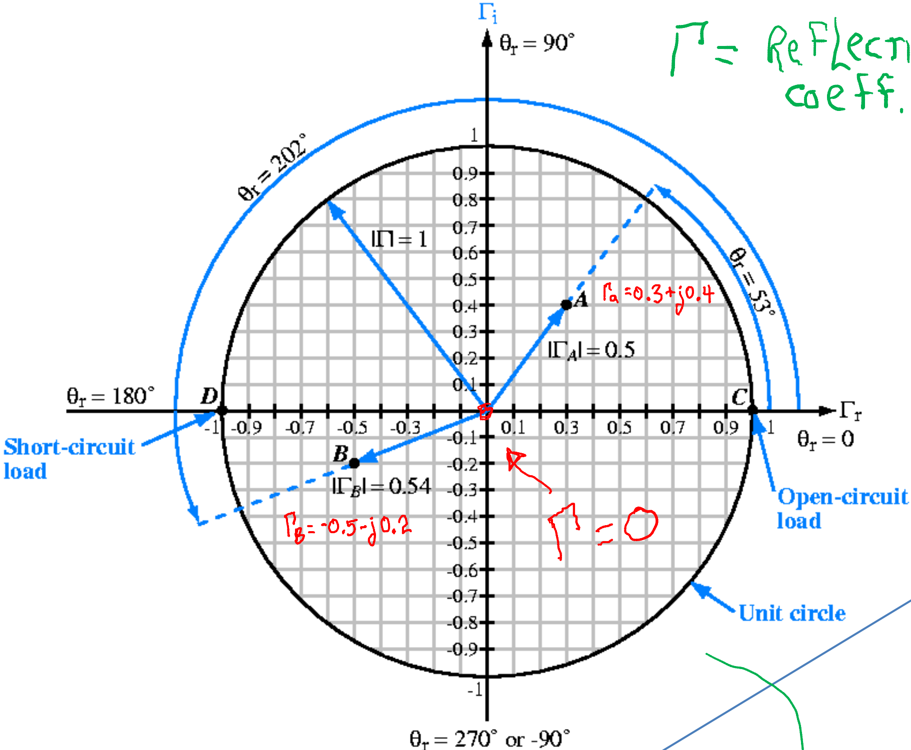
Any point within the unit circle represents certain reflection coefficient.

Smith's chart works with normalized impedances and admittances:

$$z(z) = \frac{Z(z)}{Z_0}$$



You CAN do it all with a computer of course...



Γ = Reflection coeff. $= \underbrace{|\Gamma|}_{\leq 1} e^{j\theta_R} = \Gamma_r + j\Gamma_i$

Normalized Impedance

Resistance

Reactance

$$z_L \equiv \frac{Z_L}{Z_0} = \frac{R_L + jX_L}{Z_0} \equiv r_L + jx_L$$

normalize to line impedance, Z_0

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}\left(\frac{1/Z_0}{1/Z_0}\right) = \frac{z_L - 1}{z_L + 1}$$

$$z_L = \frac{1+\Gamma}{1-\Gamma}$$

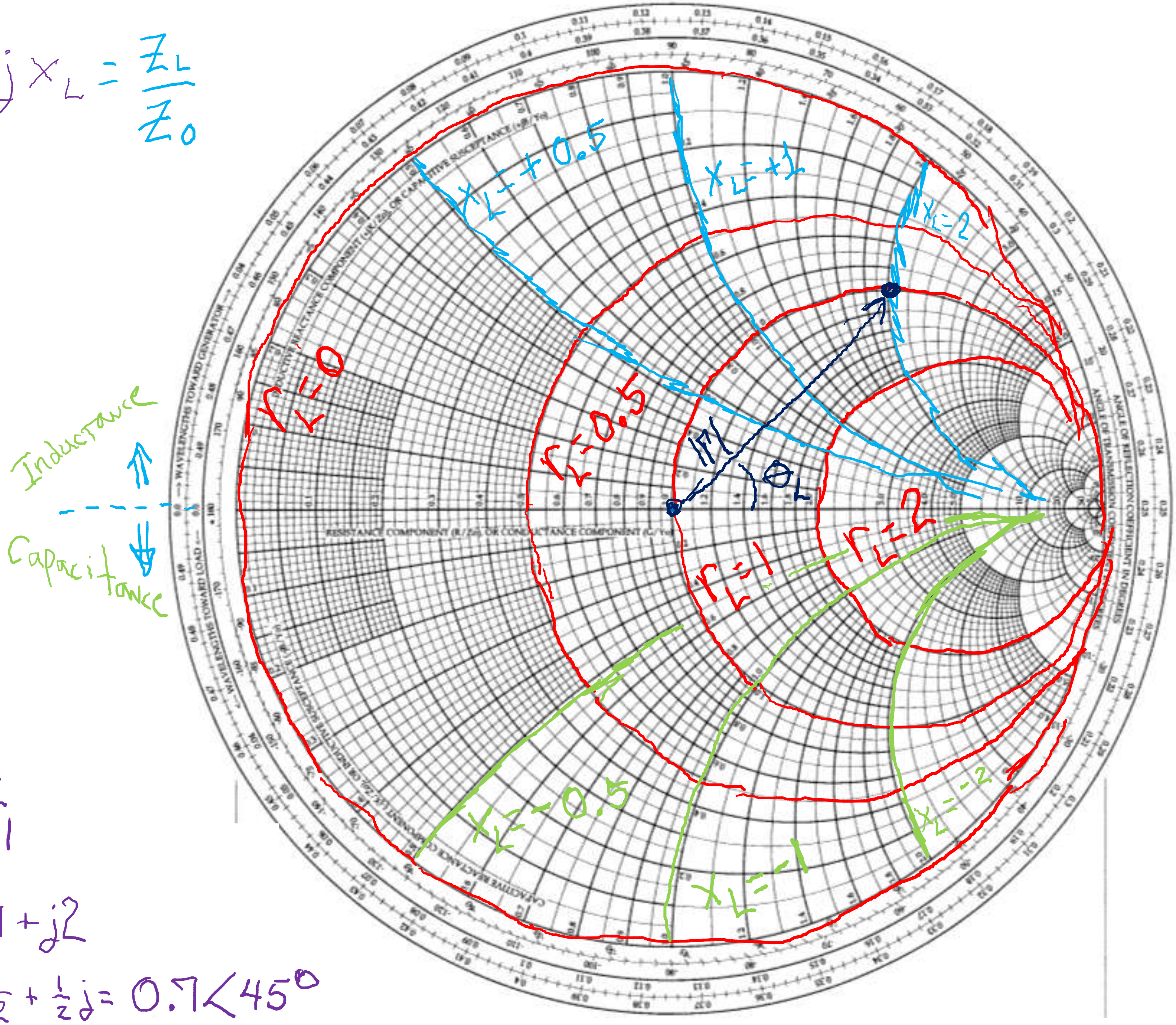
conductance

$$Y = \frac{1}{Z} = G + jB$$

Admittance

susceptance

$$z_L = r_L + j x_L = \frac{Z_L}{Z_0}$$



$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

w/ $z_L = 1 + j2$

$$\Rightarrow \Gamma = \frac{1}{2} + \frac{1}{2} j = 0.7 \angle 45^\circ$$

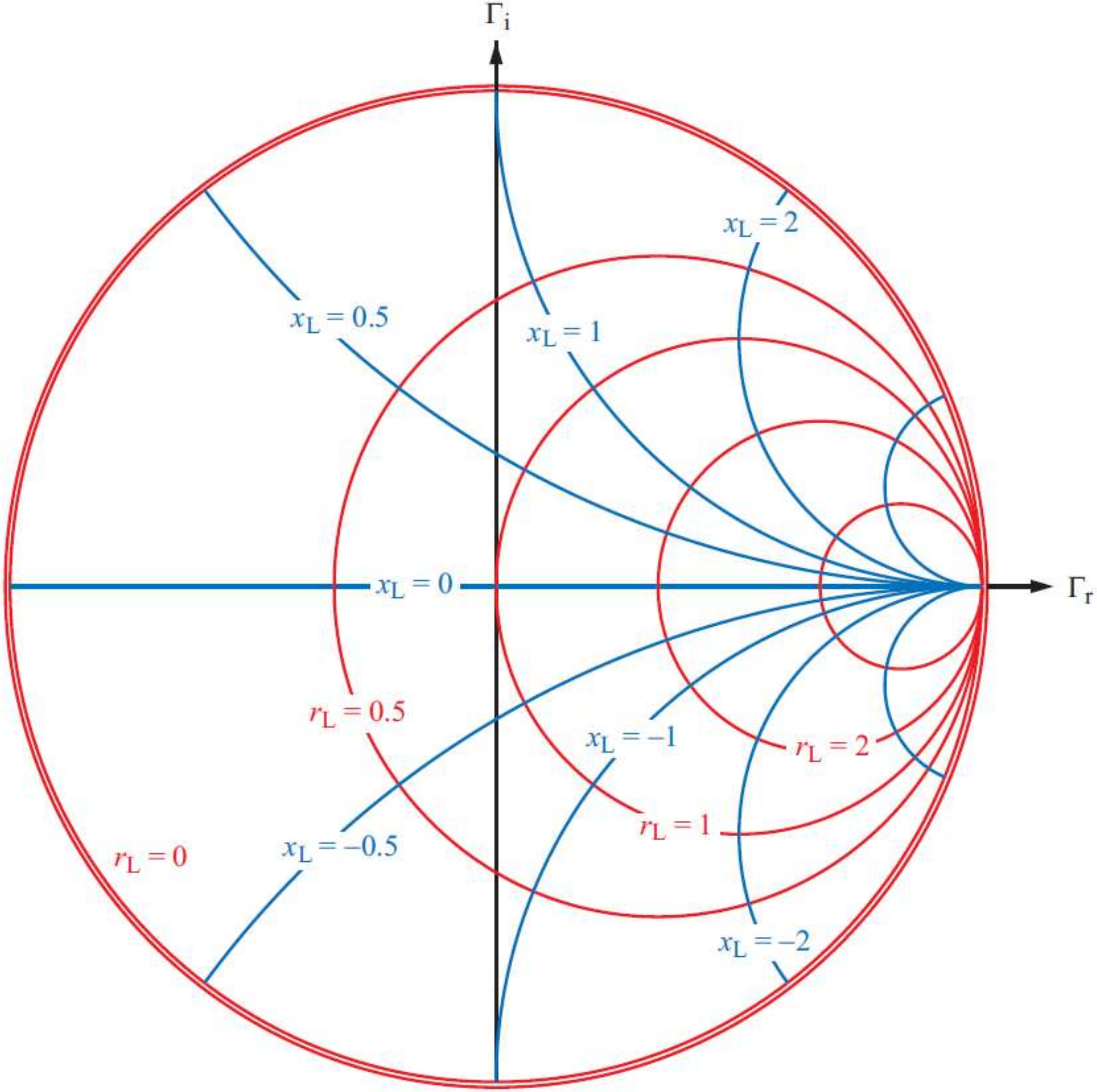


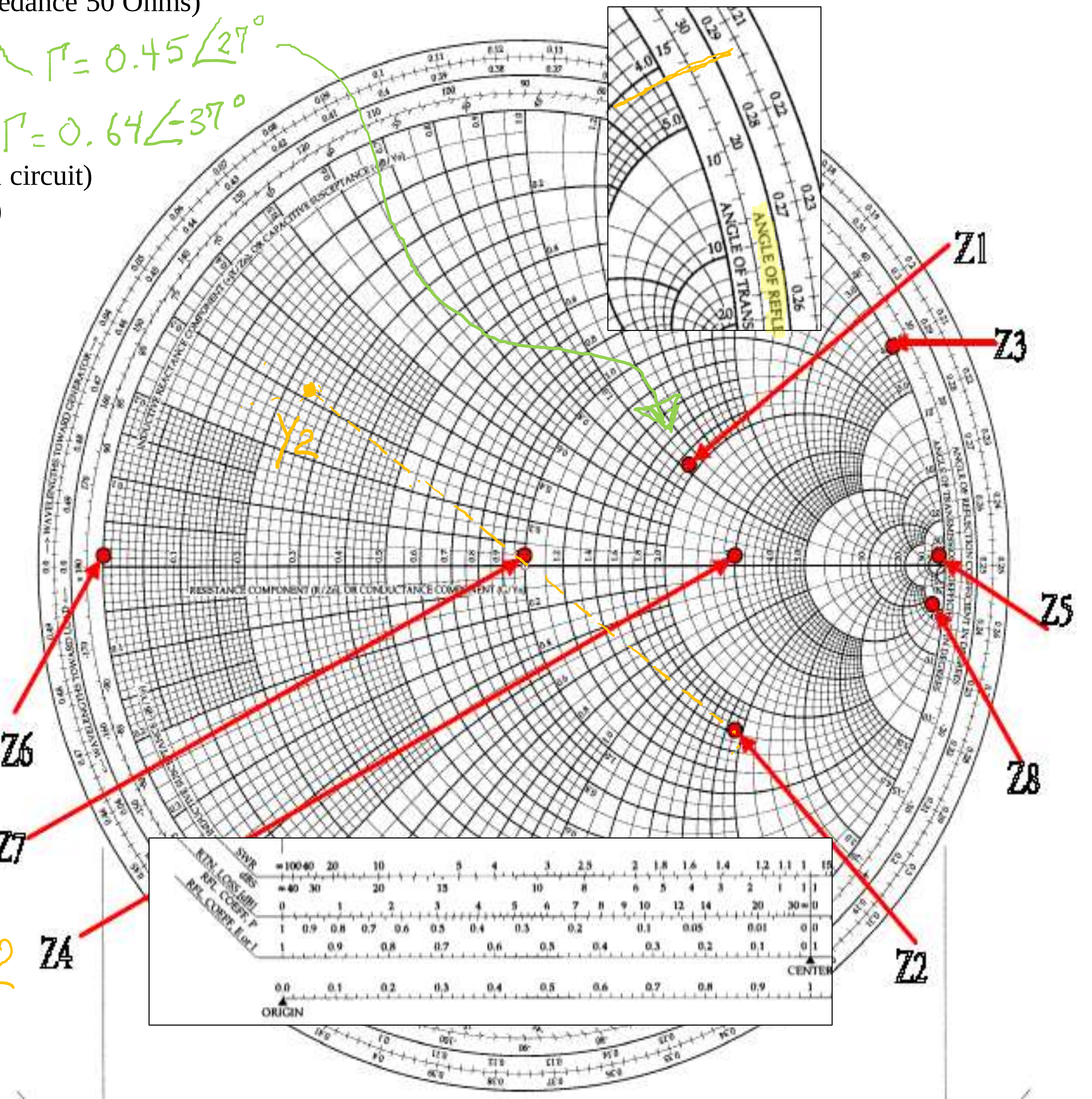
Figure 2-25: Families of r_L and x_L circles within the domain $|\Gamma| \leq 1$.

Load Impedance (with line impedance 50 Ohms)

- Z1 = 100 + j50
- Z2 = 75 -j100
- Z3 = j200
- Z4 = 150
- Z5 = infinity (an open circuit)
- Z6 = 0 (a short circuit)
- Z7 = 50
- Z8 = 184 -j900

$\Gamma = 0.45\angle 27^\circ$

$\Gamma = 0.64\angle -37^\circ$



Admittance

$y = \frac{Y}{Y_0} = g + jb$

rotate r_L around "origin" by 180° to get y_L

Ex:

$y_2 = \frac{1}{z_2} = 0.24 + j0.32$

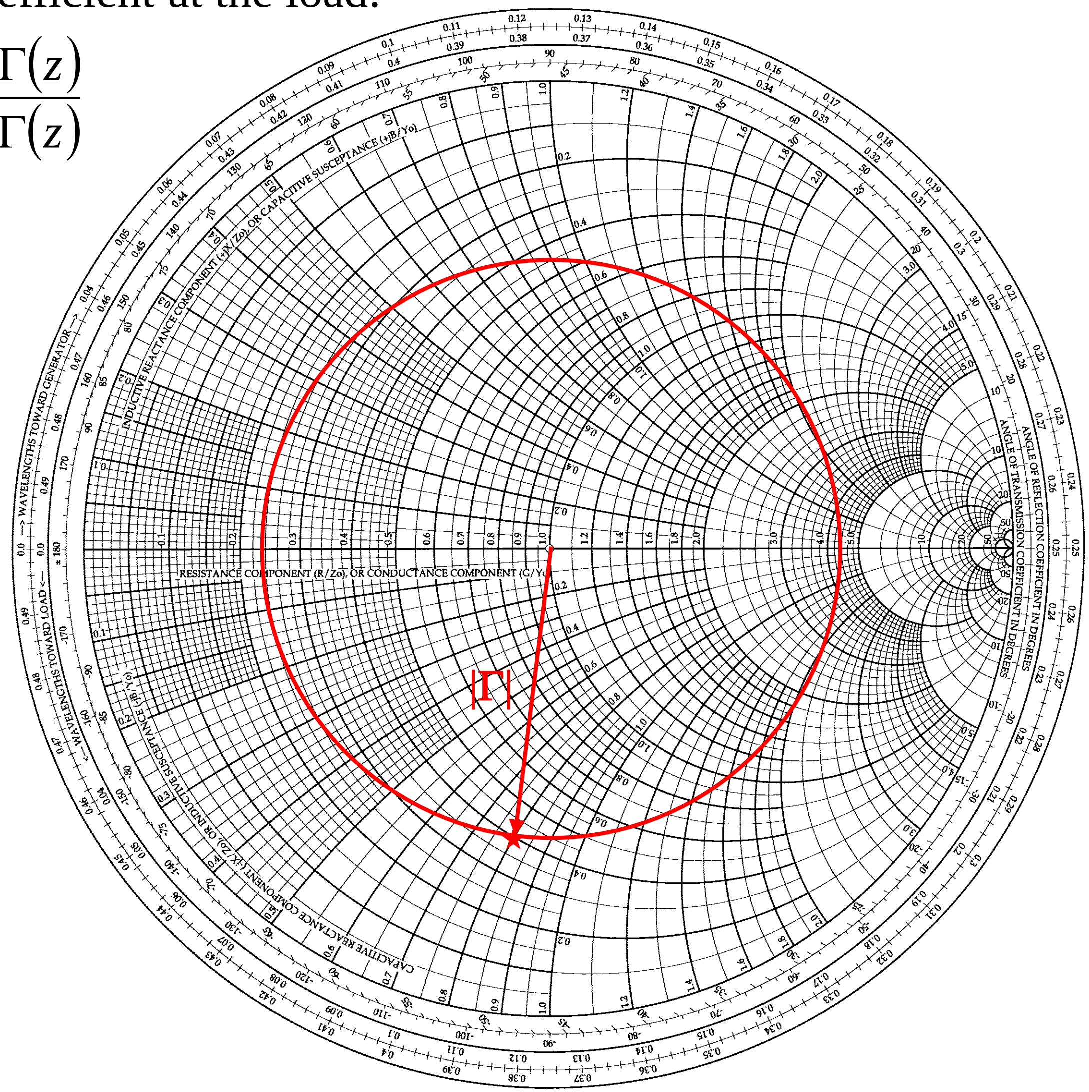
If a generalized reflection coefficient at an arbitrary point on the transmission line is defined as $\Gamma(z) = |\Gamma| e^{j(\theta_r + 2\beta z)}$, then the normalized impedance at that point can be expressed through the generalized reflection coefficient the same way the normalized load impedance is expressed in terms of the reflection coefficient at the load:

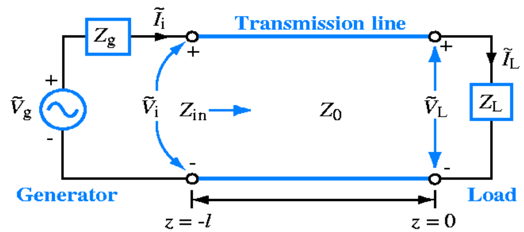
$$z(z) = \frac{Z(z)}{Z_0} = \frac{1+\Gamma e^{-j2\beta z}}{1-\Gamma e^{-j2\beta z}} = \frac{1+\Gamma(z)}{1-\Gamma(z)}$$

This observation suggests that Smith's chart can be used to represent the impedance at any point on the line.

The reflection coefficient along a particular transmission line maintains its magnitude; only its phase changes. Therefore, the normalized impedance at any point on the transmission line must lie on a circle with radius $|\Gamma|$.

Also, it is obvious that the impedance on the line repeats itself every $\lambda/2$ length of the line. Therefore, the Smith chart represents a $\lambda/2$-long section of a transmission line.

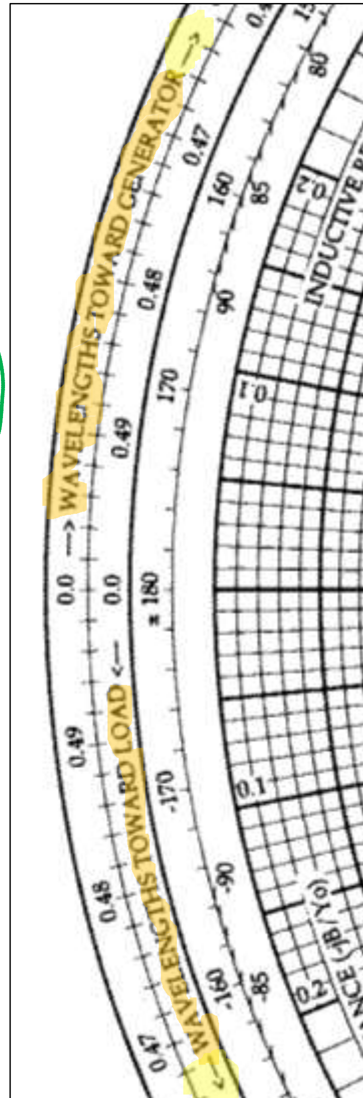


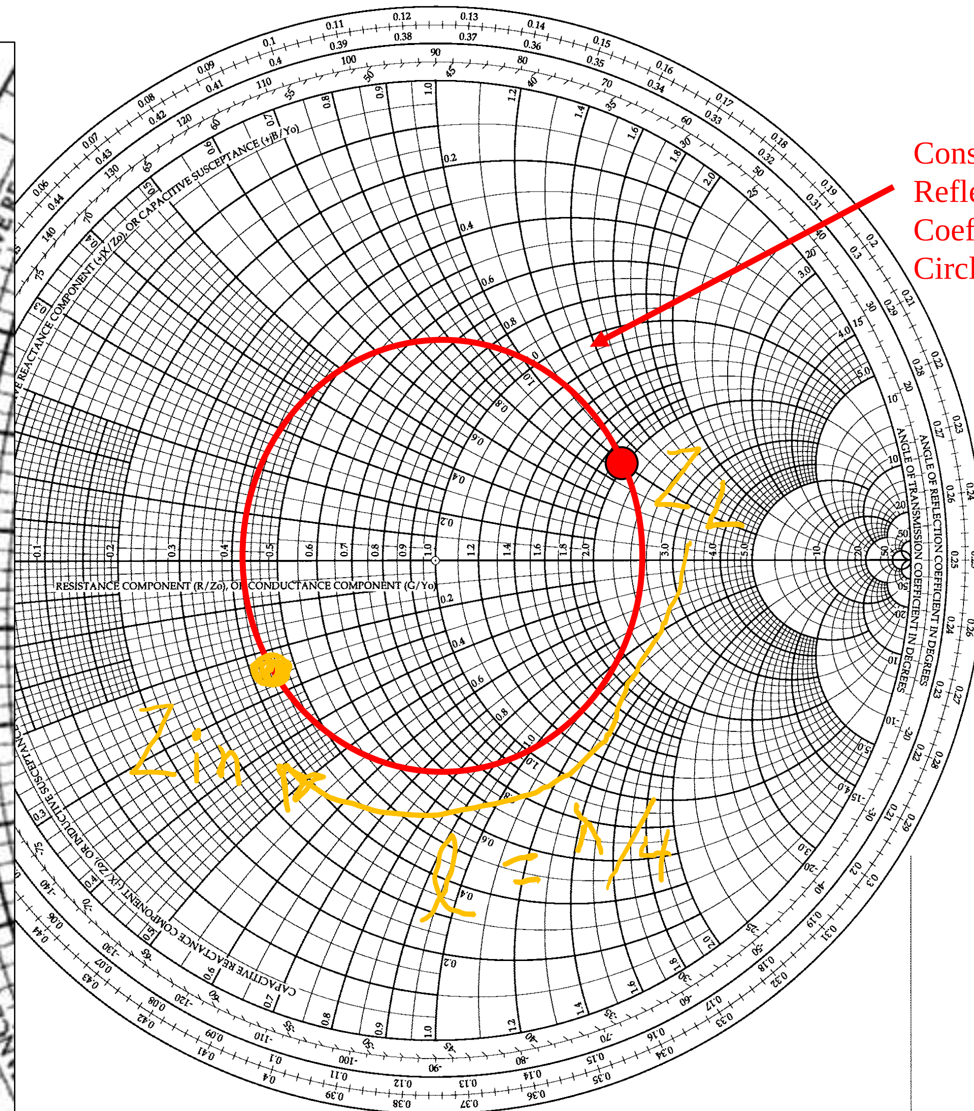


$$z_{in} = \frac{Z_{in}}{Z_0} = \frac{1+\Gamma e^{-j2\beta l}}{1-\Gamma e^{+j2\beta l}}, \quad w/\ \beta = \frac{2\pi}{\lambda} \leftarrow \text{wavelength}$$

Map Z_L around clockwise on constant $|\Gamma|$ contour by $\frac{l}{\lambda}$ to get

Z_L





Constant Reflection Coefficient Circle

Full Circle is One Half Wavelength Since Everything Repeats

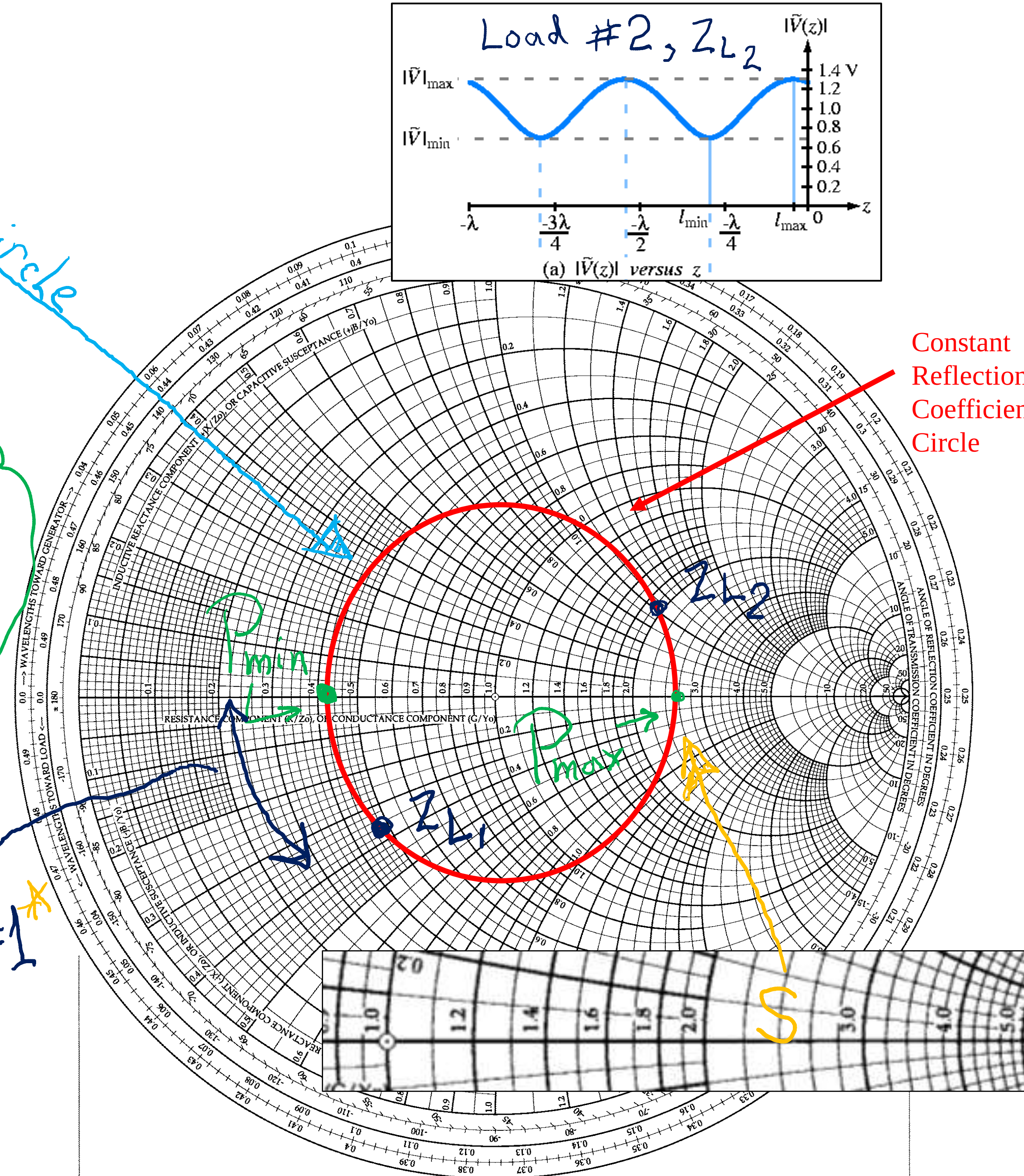
Standing Wave Ratio (SWR)

$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

SWR circle

P_{max} & P_{min} are points, not power

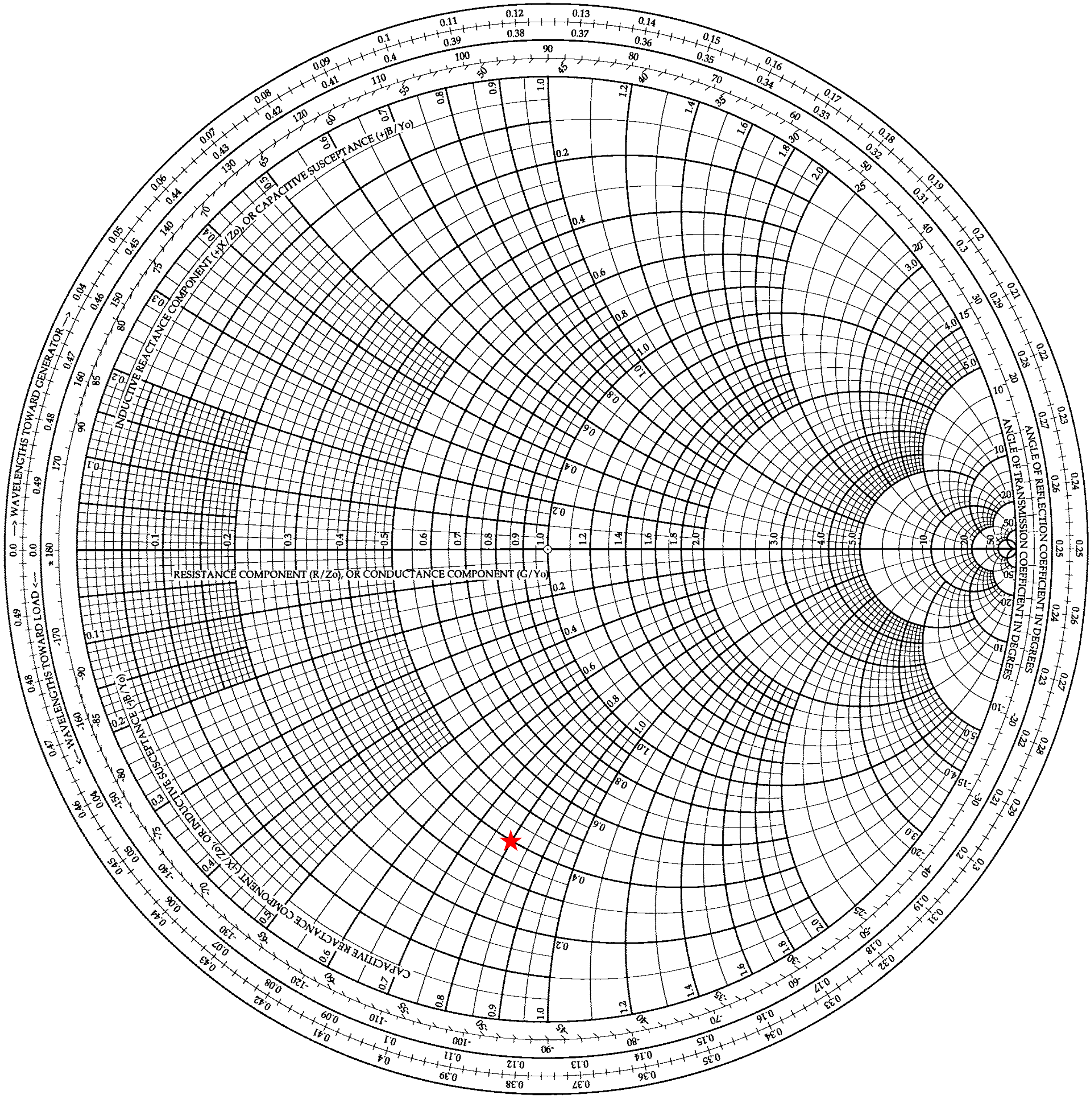
wavelengths to 1st minimum from Load #1*



Full Circle is One Half Wavelength Since Everything Repeats

* Two independent examples

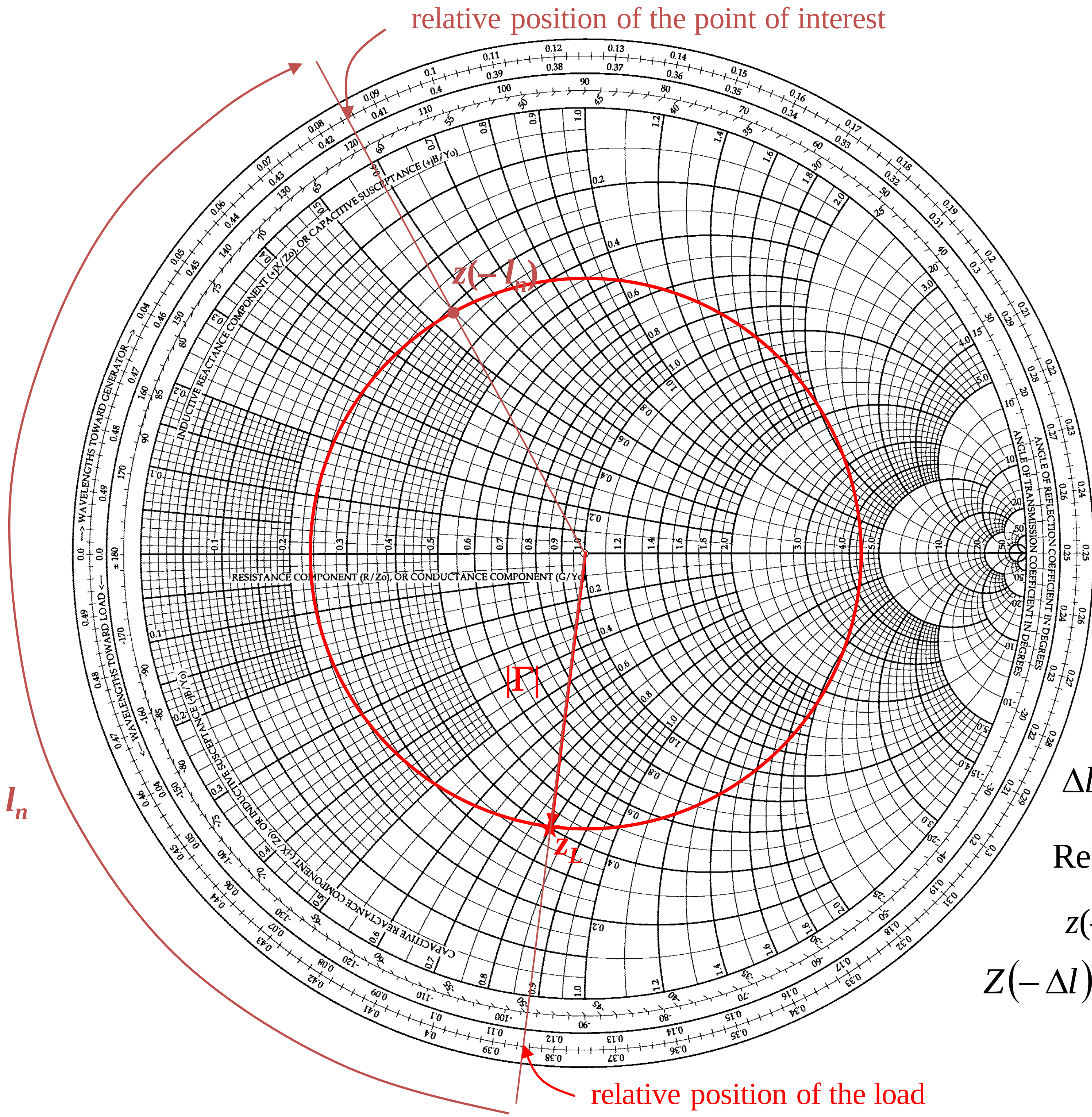
Smith chart actually makes problems easier where you want a specific Z_{in} !!!!!



__Examples:__

Determine the load impedance denoted by the red star on the chart if this load terminates a 50-Ω transmission line.

The point represents
$z_L = 0.4 - j0.8$
Then
$$Z_L = z_L Z_0 = 20 - j40\ \Omega$$



Example:

Consider the load impedance from the previous example. Determine the impedance 20 cm away from the load if the signal wavelength on the line is 1 m.

$z_L = 0.4 - j0.8$

Rel. pos. load = 0.385

$$\Delta l = 20\,\text{cm} \Rightarrow l_n = \frac{\Delta l}{\lambda} = \frac{0.2}{1} = 0.2$$

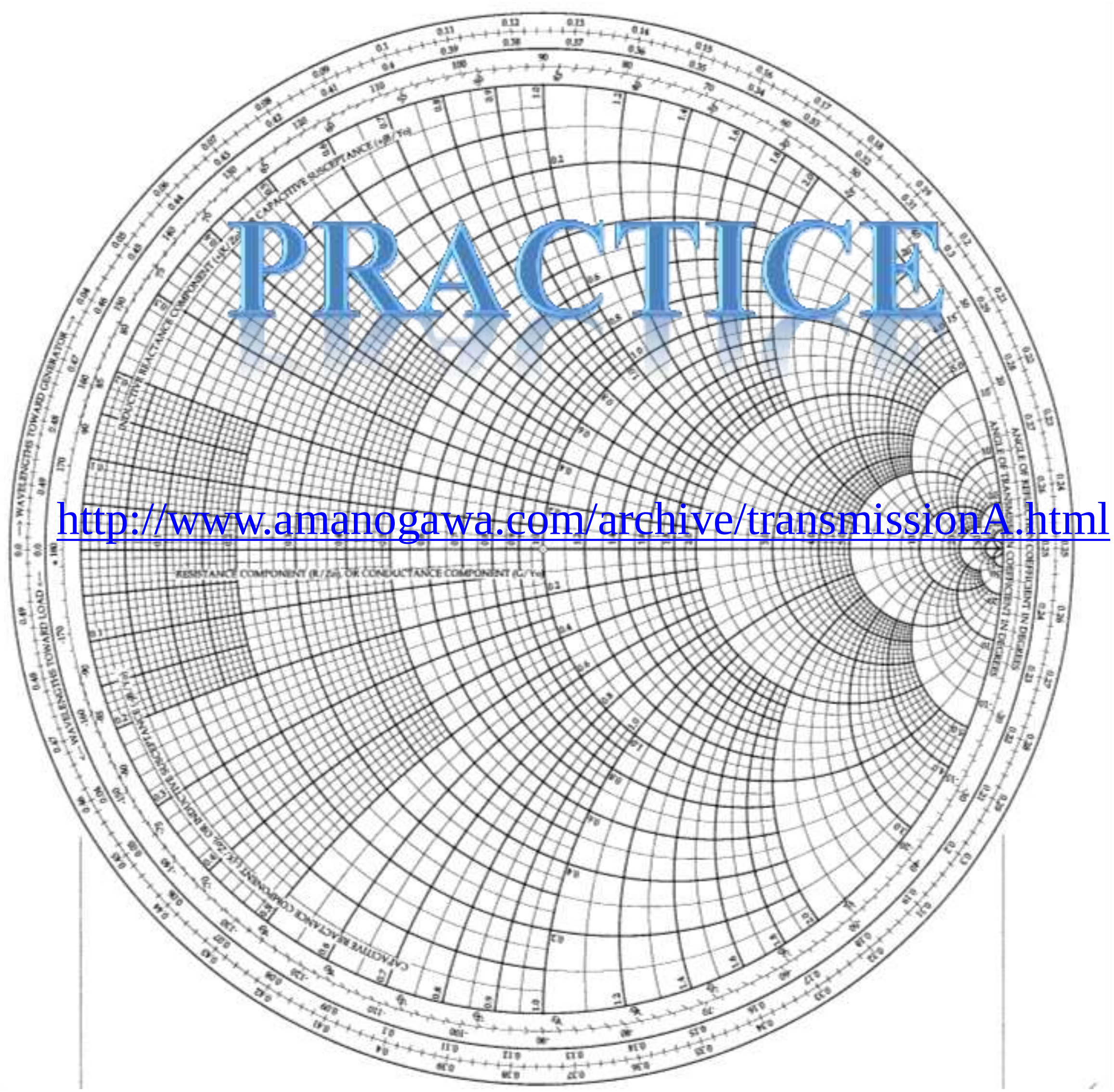
Rel. pos. z = 0.385 + 0.2 = 0.585

$z(-l_n) = 0.31 + j0.55$

$$Z(-\Delta l) = Z_0 z(-l_n) = 15.5 + j27.5\,\Omega$$

http://en.wikipedia.org/wiki/Smith_chart

http://www.microwaves101.com/encyclopedia/smithchart.cfm



http://www.amanogawa.com/archive/transmissionA.html

Example 2-11: Smith Chart Calculations

A 50-Ω lossless transmission line of length 3.3λ is terminated by a load impedance $Z_L = (25 + j50)\ \Omega$.

$$z_L = \frac{Z_L}{Z_0} = \frac{25 + j50}{50} = 0.5 + j1$$

(a)

$$d_{max} = (0.25 - 0.135)\lambda = 0.115\lambda$$

$$d_{min} = (0.5 - 0.135)\lambda = 0.365\lambda$$

(b)

$$z_{in} = 0.28 - j0.40$$

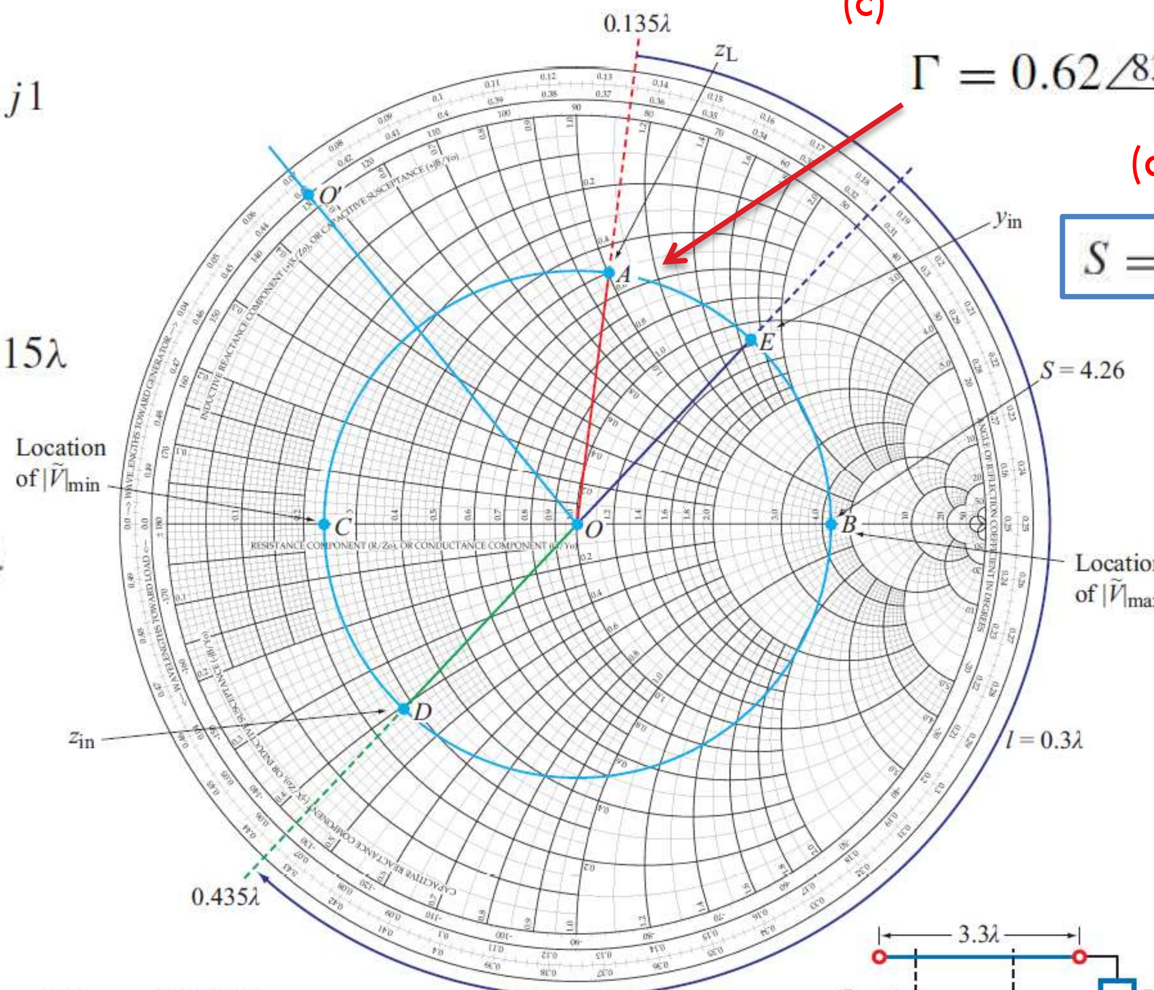
$$Z_{in} = z_{in} Z_0 = (0.28 - j0.40)50 = (14 - j20)\ \Omega$$

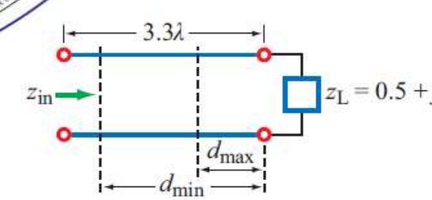
(c)

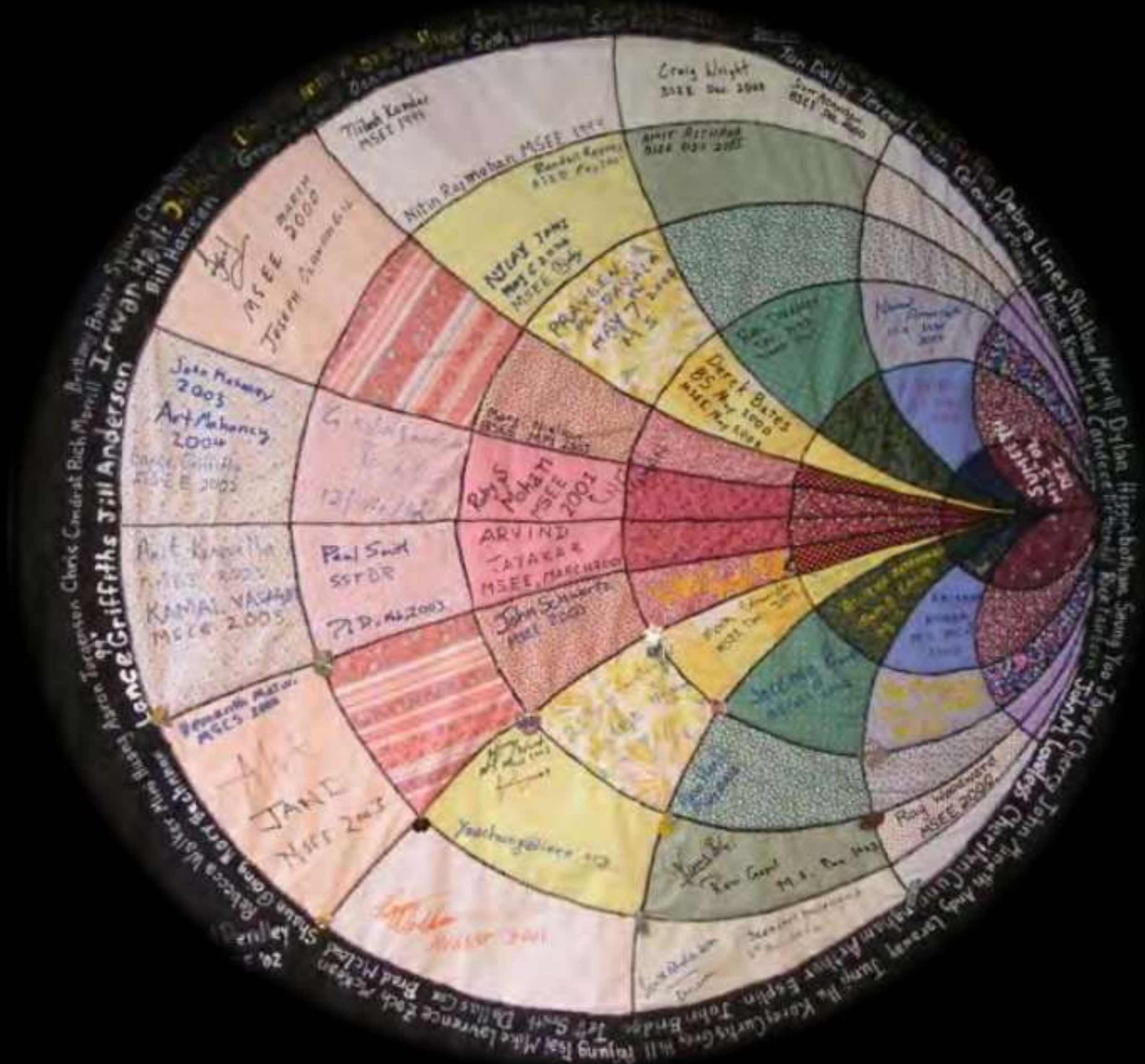
$$\Gamma = 0.62\angle 83^\circ$$

(d)

$$\boxed{S = 4.26}$$

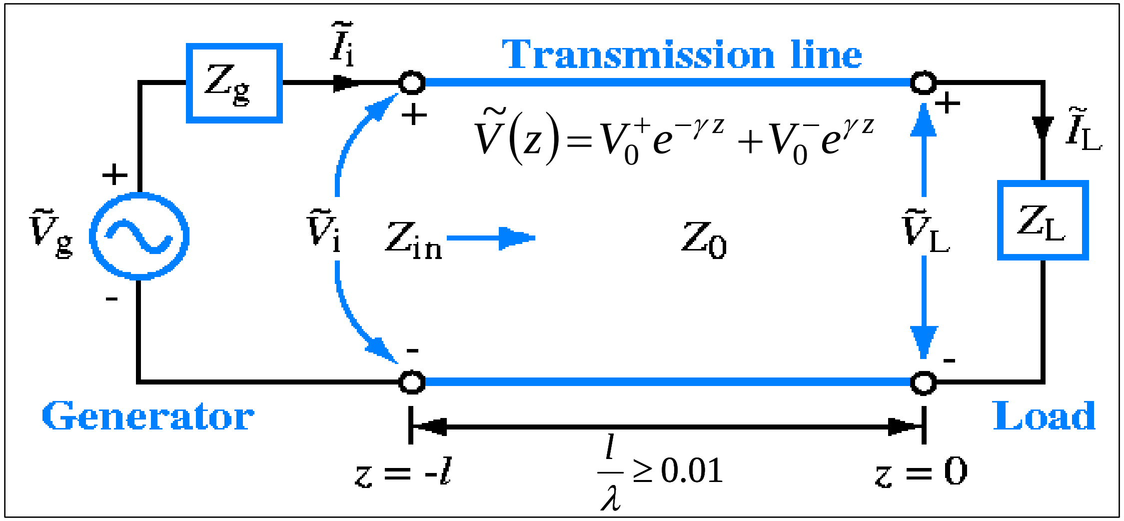






This quilt was made by Cynthia Furse, a Professor in the Department of Electrical and Computer Engineering at Utah State University. She has her students sign when they have finished their MS or BS design projects.

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\varepsilon_r}} = f\lambda$$



$$\lambda = \frac{2\pi}{\beta}$$

$$v(z,t) = |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \phi^+) + |V_0^-| e^{\alpha z} \cos(\omega t + \beta z + \phi^-)$$

$$\gamma = \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

$$\frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = Z_o = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

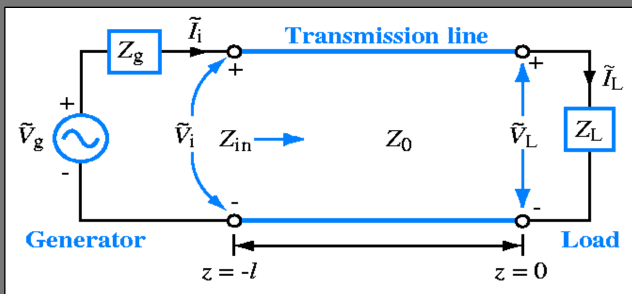
$$S = \frac{|\tilde{V}(z)|_{\max}}{|\tilde{V}(z)|_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

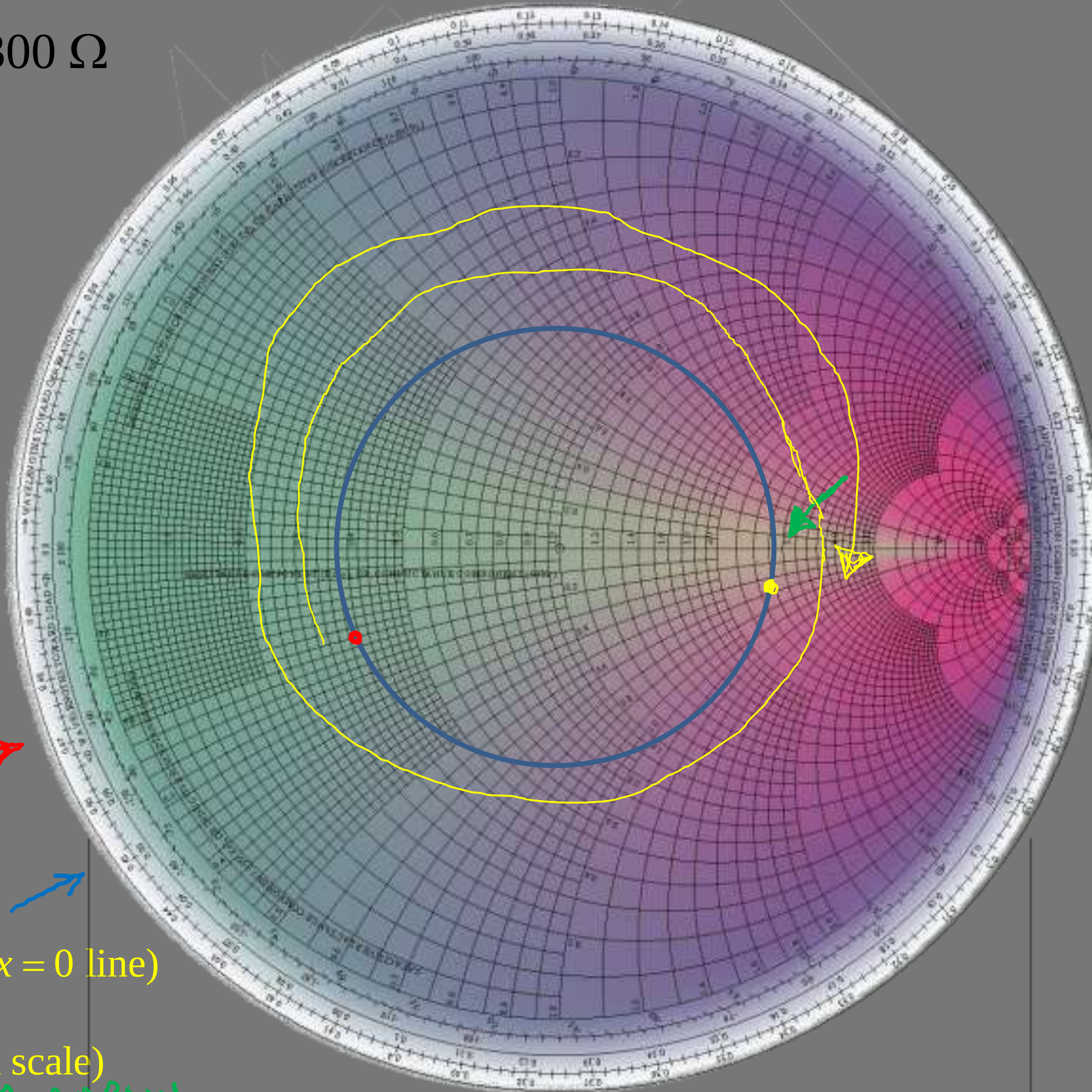
$$Z_{in} = Z(-l) = \frac{\tilde{V}(-l)}{\tilde{I}(-l)} = Z_0 \left[\frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}}\right] = Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}\right]$$

Given: $Z_L = (120 - j60)\ \Omega$, $Z_0 = 300\ \Omega$

and $\ell = 0.8\lambda$

Find: Z_{in}, Γ, VSWR





1) normalize: $z_L = \dfrac{120 - j60}{300} = 0.40 - j0.20$

2) locate point: (to lower left of center)

on $r_L = 0.40$ circle and $x_L = -0.20$ curve

3) $\Gamma = \dfrac{z_L - 1}{z_L + 1} = 0.45\angle -153^\circ$

Radius of circle

(can verify $|\Gamma|$ on RFL COEFF scale or $x = 0$ line)

4) $VSWR = \dfrac{1+|\Gamma|}{1-|\Gamma|} = \dfrac{1.45}{0.55} = 2.6$ (or by SWR scale)

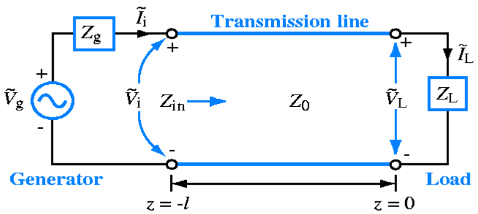
see green arrow

5) rotate Γ_L $2\beta l = 2(288^\circ) = 576^\circ = (360^\circ + 216^\circ)$ ← one time around Smith Chart is 0.5 λ

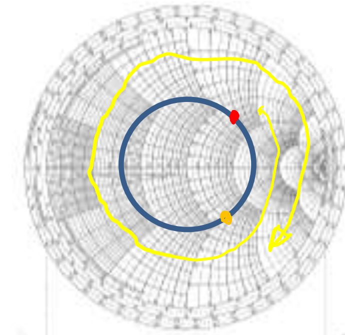
to get $\Gamma_{in} = |\Gamma| e^{j(\phi - 2\beta l)} \Rightarrow z_{in} = 2.52 - j0.46$ (to lower right of center) → Yellow Dot

6) Un-normalize: $Z_{in} = (2.52 - j0.46)(300\ \Omega) = (755 - j138)\ \Omega$

SUNDRIES



- what does a complex Γ actually mean?
- what does "Γ" changing as it moves toward the generator mean?



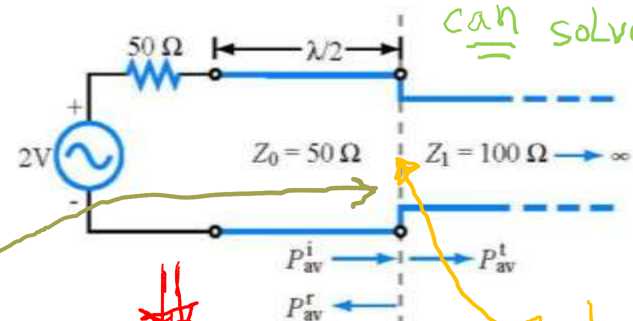
$$\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right) \neq \frac{1}{\tan^{-1}(x)}$$

,

$$\cot(\underbrace{\beta l}_{\text{in Radians}})$$

There is often more than one way to solve these problems!

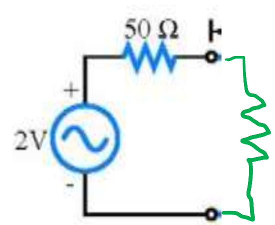
can solve using V_0^+, etc.



$\Gamma = \frac{1}{3}$

why?

Q: Does this situation ever happen?



$Z_{in} = 100\,\Omega$

why?

Power "absorbed" by Z_{in} equals P_{av}^t

why?

(Note: this takes into account multiple bounces!)

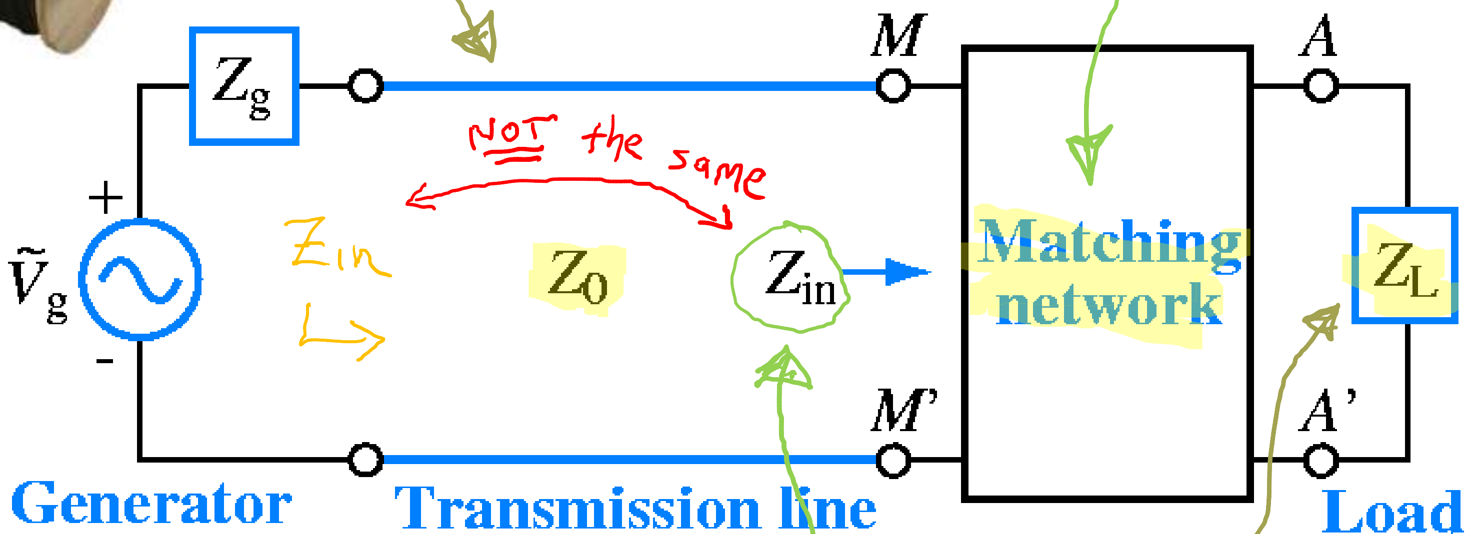
$$\left.\begin{aligned} &P_{AV}^i - P_{AV}^r = P_{AV}^t \\ \text{and } &P_{AV}^r = |\Gamma|^2 P_{AV}^i \end{aligned}\right\} \text{Why?}$$

Impedance Matching

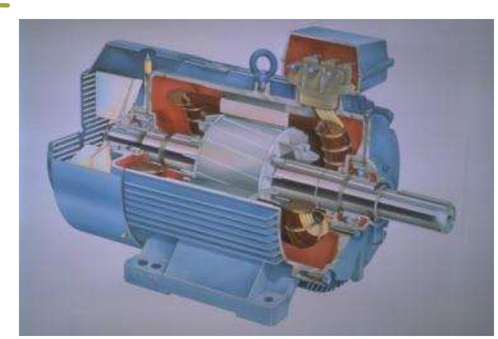
May not be able to control Z_o

Ideally, this will be purely reactive

Why?

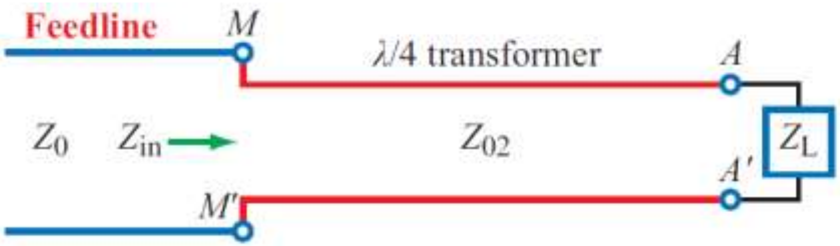


May not have control over Z_L

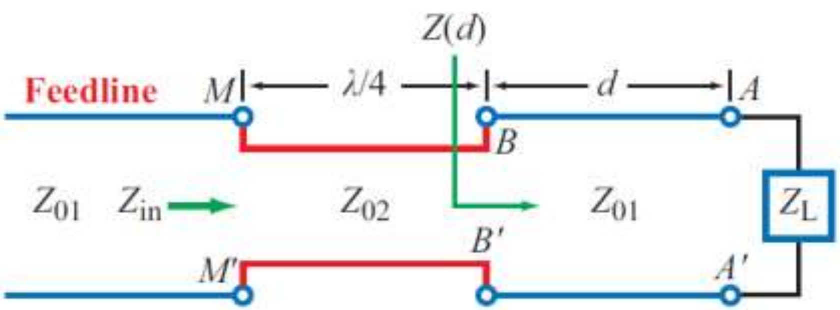


Use a matching network to make this equal Z_o

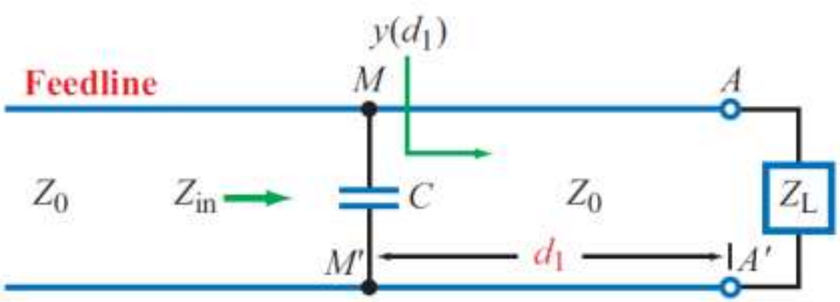
Examples of Matching Networks



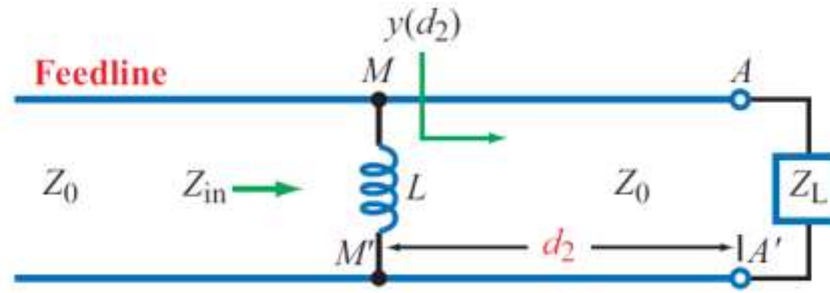
(a) In-series $\lambda/4$ transformer inserted at AA'



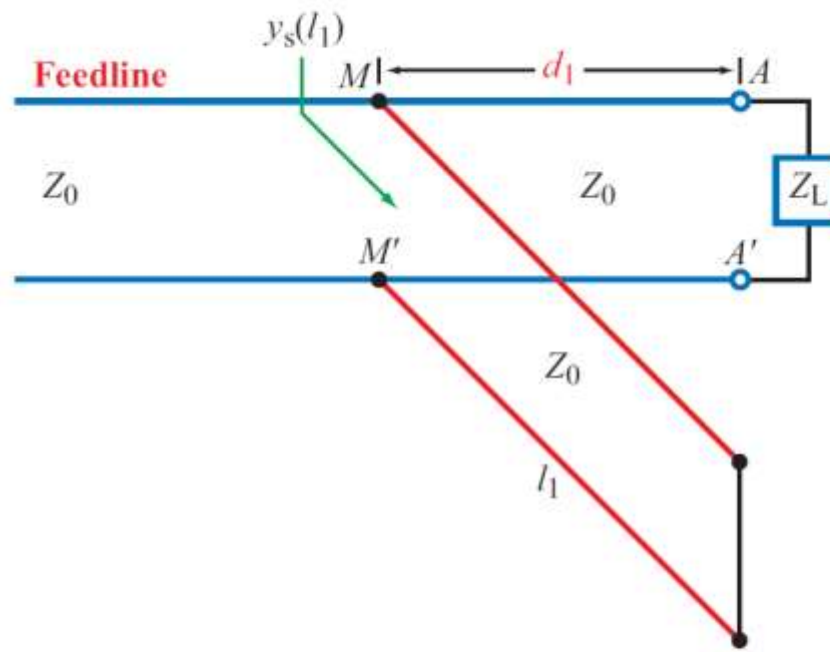
(b) In-series $\lambda/4$ transformer inserted at $d = d_{max}$ or $d = d_{min}$



(c) In-parallel insertion of capacitor at distance d_1



(d) In-parallel insertion of inductor at distance d_2



(e) In-parallel insertion of a short-circuited stub

Single-Stub Impedance Matching

The power absorbed by the load is maximum (100% of the incident power) when the transmission line is matched, i.e., the load impedance equals the characteristic impedance, $Z_L = Z_0$.

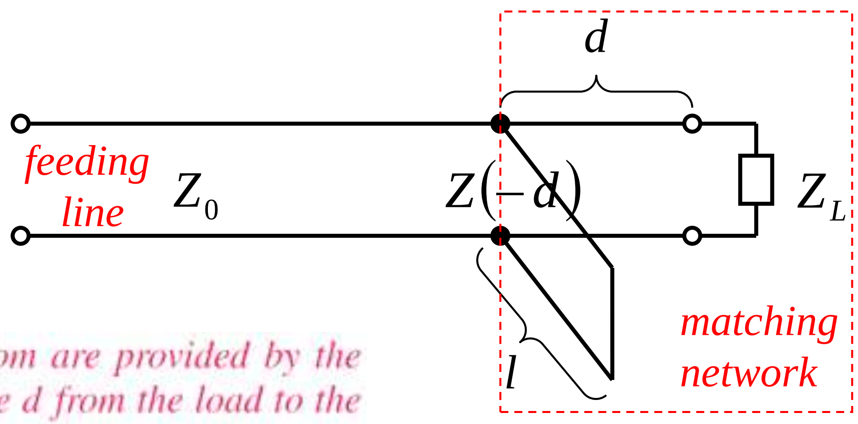
Unfortunately, often $Z_L \neq Z_0$

Fortunately, there are means to match the load to the transmission line via impedance-matching network.

One of the possibilities is:

Single-stub matching using a short-circuited section of a transmission line.

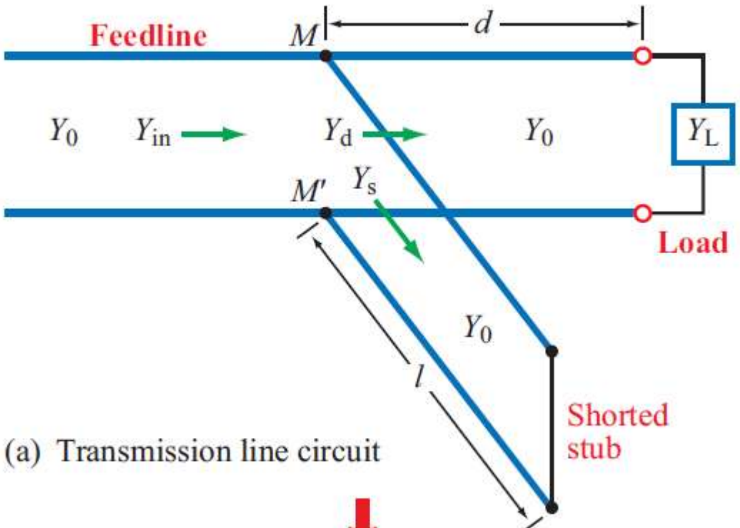
Essentially, a short-circuited section of the same type transmission line as the feeding (main) line is connected in parallel to the main line at a position that is relatively close to the load. The point of connection and the length of the short-circuited stub can be chosen in such a way so that to ensure an input impedance $Z(-d) = Z_0$ of the matching network that contains the load.



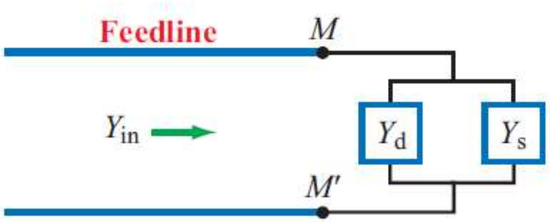
The required two degrees of freedom are provided by the length l of the stub and the distance d from the load to the stub position.

Example 2-14: Single-Stub Matching

Repeat Example 2-13, but use a shorted stub (instead of a lumped element) to match the load impedance $Z_L = (25 - j50)\ \Omega$ to the 50-Ω transmission line.



(a) Transmission line circuit



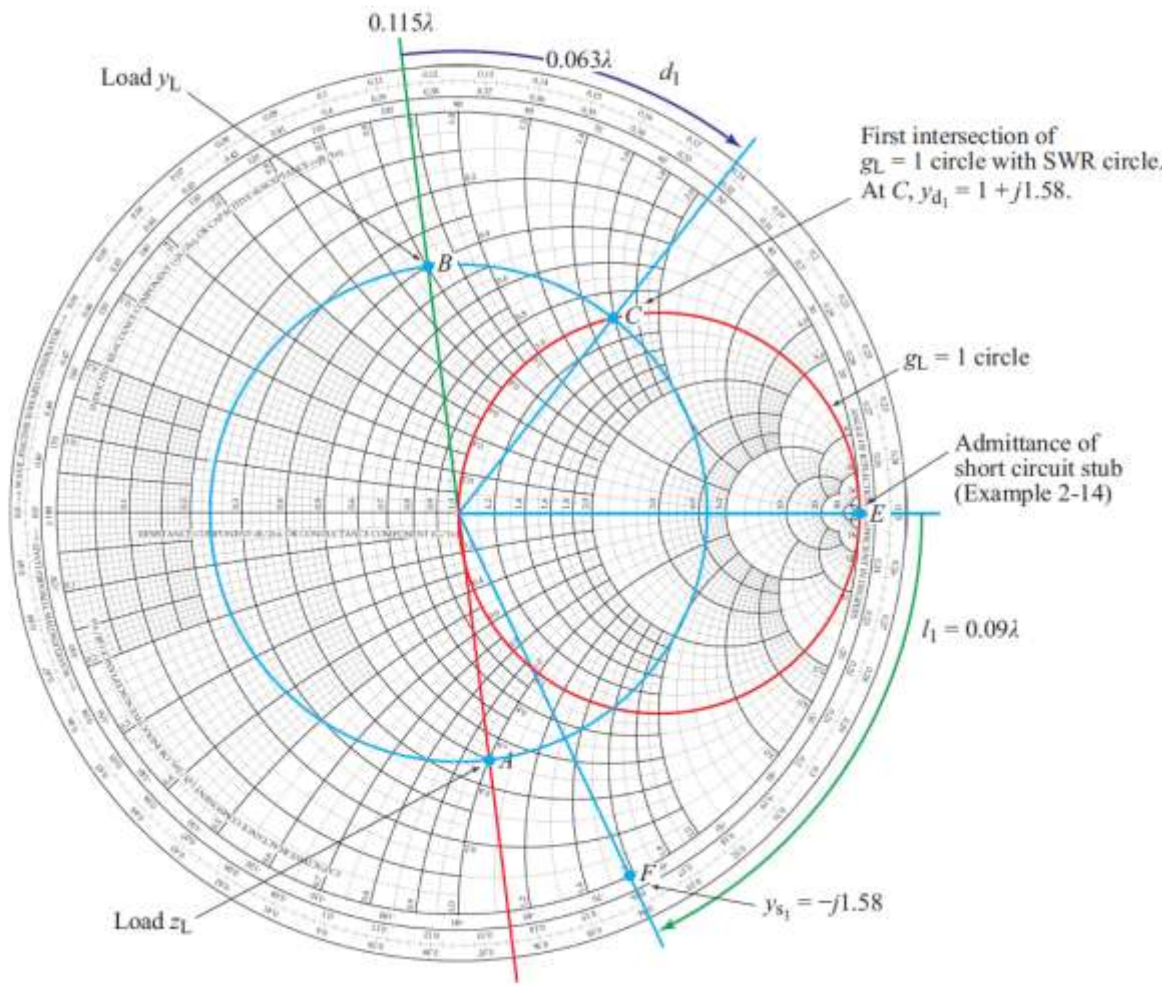
(b) Equivalent circuit

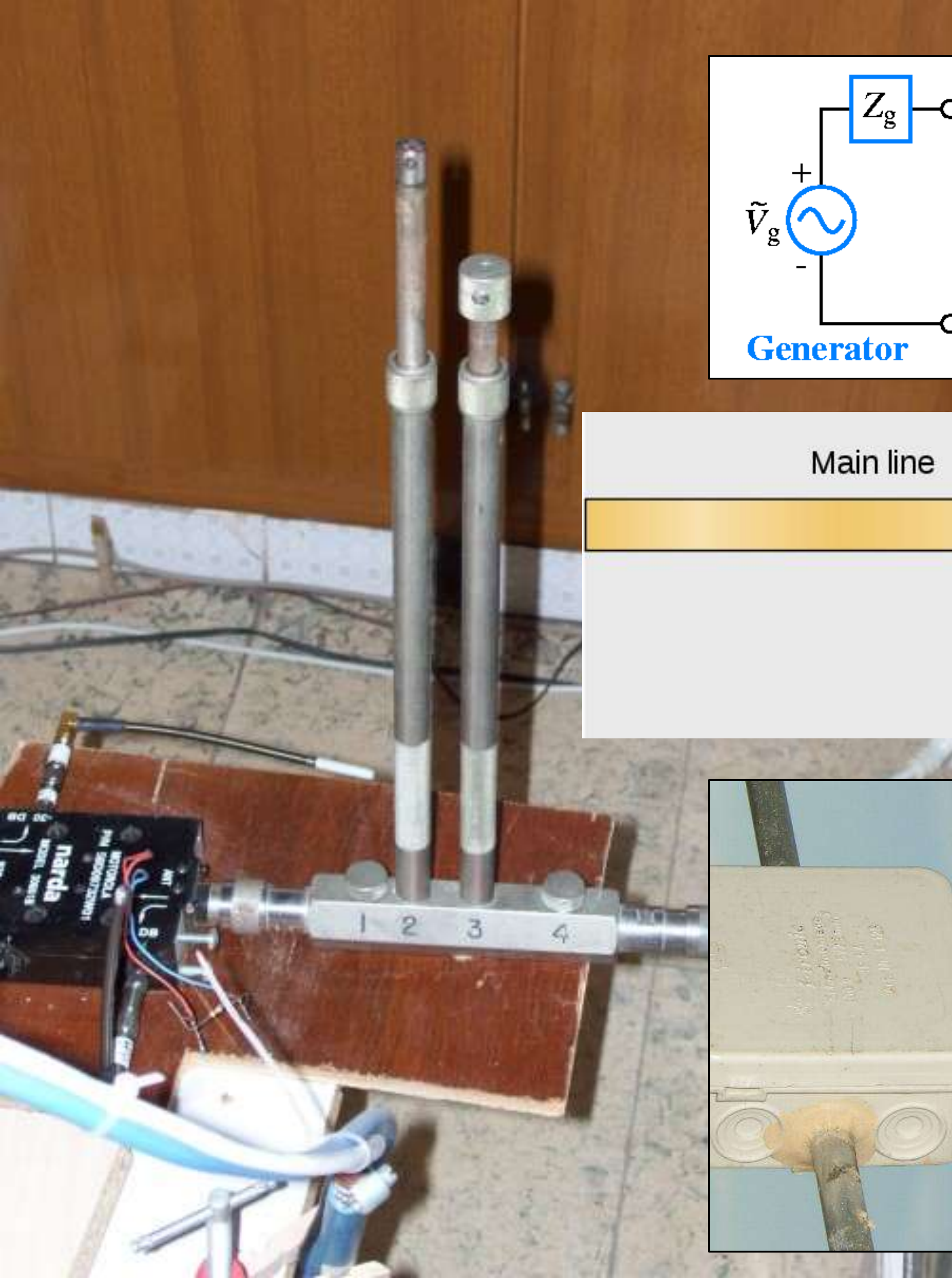
Solution: In Example 2-13, we demonstrated that the load can be matched to the line via either of two solutions:

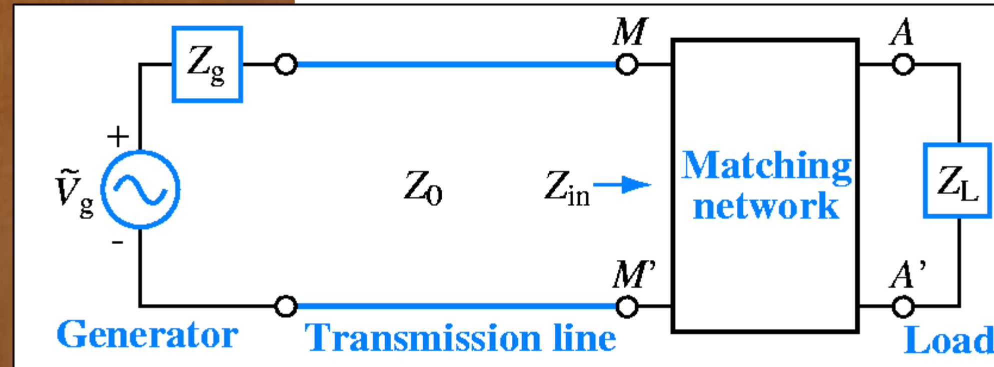
(1) $\boxed{d_1 = 0.063\lambda,}$ and $y_{s_1} = jb_{s_1} = -j1.58,$

(2) $d_2 = 0.207\lambda,$ and $y_{s_2} = jb_{s_2} = j1.58.$

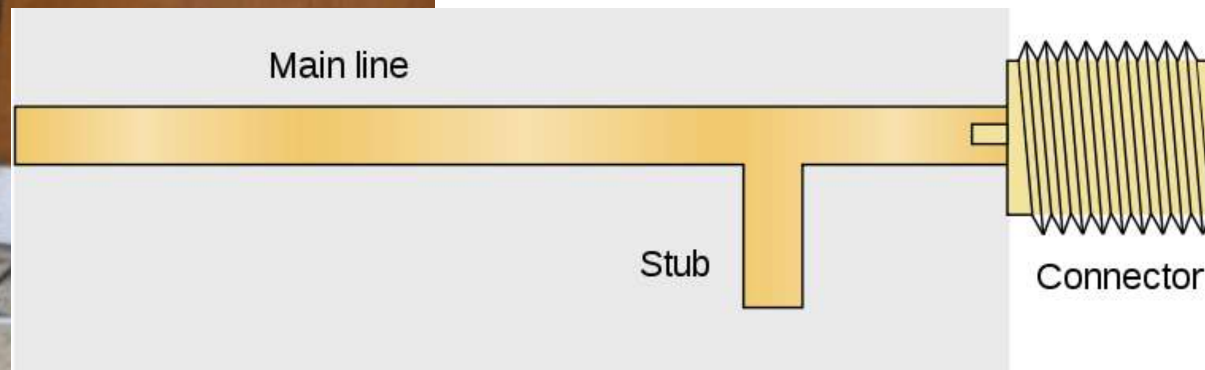
$$\boxed{l_1 = (0.34 - 0.25)\lambda = 0.09\lambda}$$



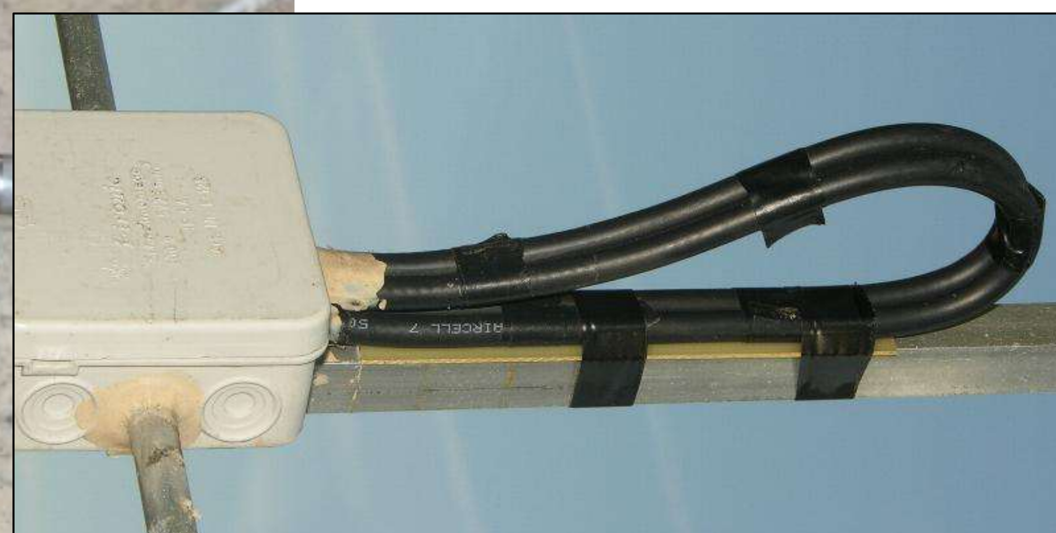


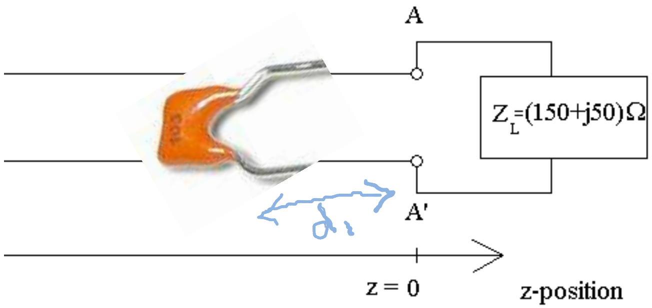


Z_g
$\tilde{V}_g$
+
-
M
M'
A
A'
Z_0
Z_{in}
Matching network
Z_L
Generator
Transmission line
Load

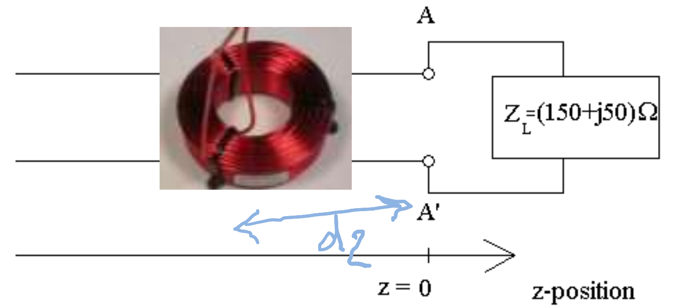


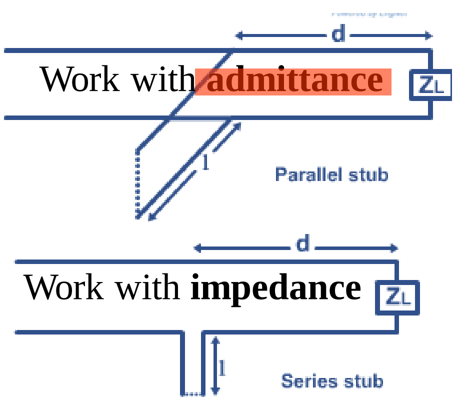
Main line
Stub
Connector





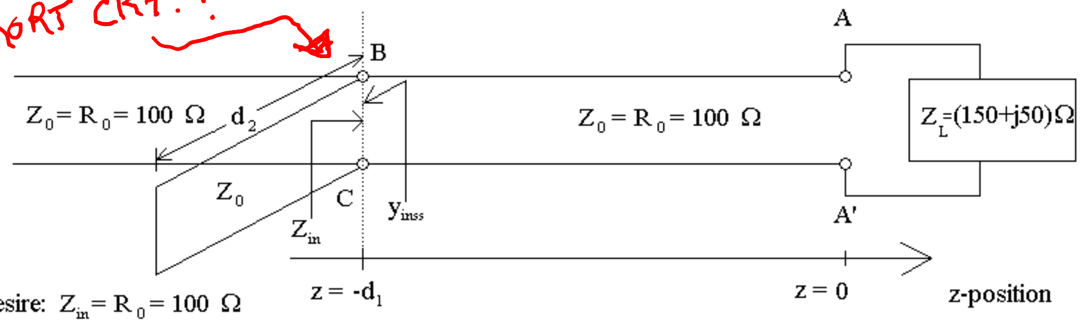
OR
=





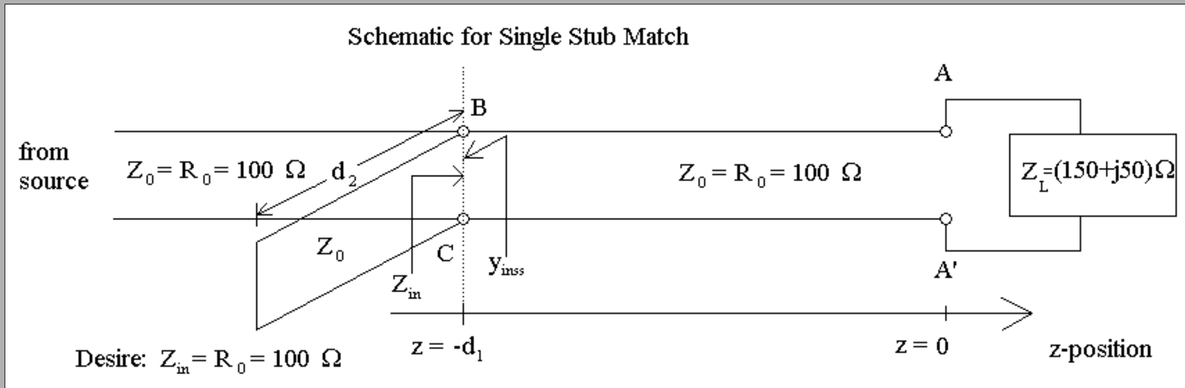
Why isn't this a short ckt.?

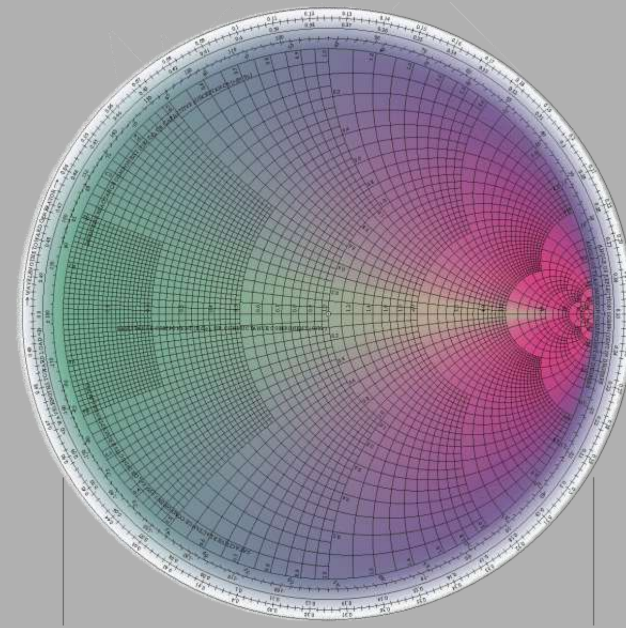
Schematic for Single Stub Match



Quarter-Wave Transformer

design $\rightarrow = \sqrt{Z_B \cdot Z_0}$





1) Normalize the load: $z_L = \dfrac{150 + j50}{100} = 1.5 + j0.50$

2) Convert to admittance: $y_L = 1/z_L = 0.60 - j0.20$

Note: $z_{in}(-l = -\lambda/4) = \dfrac{1+\Gamma e^{-j2\beta(\lambda/4)}}{1-\Gamma e^{-j2\beta(\lambda/4)}} = \dfrac{1+\Gamma e^{-j\pi}}{1-\Gamma e^{-j\pi}}$

$$= \frac{1-\Gamma}{1+\Gamma} = \frac{1}{z_{in}(0)} = \frac{1}{z_L} = y_L$$

$\therefore$ going halfway ($\lambda/4$) around the smith chart converts any impedance to its reciprocal (admittance)!

3) Find the intersection between $|\Gamma_L| = 0.27$ ($VSWR = 1.78$) circle and the $g = 1$ circle

Note that two intersection points exist, but one minimizes the length d_1. In terms of admittance

$\Rightarrow y_B = 1.0 + j0.58 = g_B + jb_B$

$\therefore\ d_1 = z_B - z_{A'} = (0.647 - 0.454)\lambda = 0.193\lambda$

4) Find the minimum length of a shorted stub, that when inserted in parallel at B, produces a match ($Z_{in} = Z_B = R_0$).

$y_{ss} = \dfrac{1}{z_{ss}} = \infty$ (exists at the right extreme)

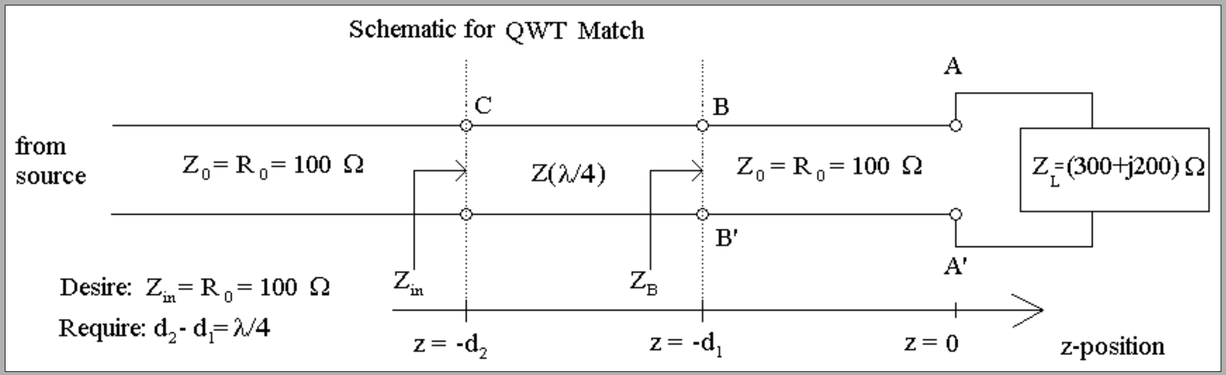
Desire: $y_{inss} = y_C = 0 - jb_B = -j0.58$

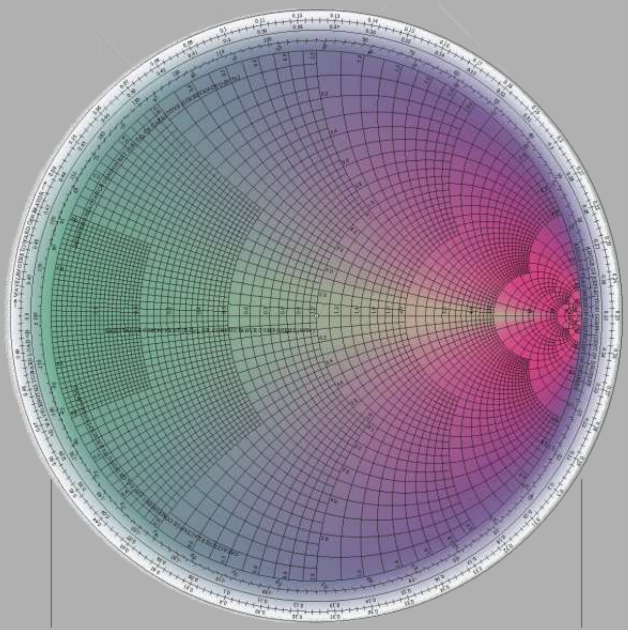
This occurs at the pos. on the S.C.: $z_C = 0.416\lambda$

$\therefore d_2 = z_C - z_{ss} = (0.416 - 0.250)\lambda = 0.166\lambda$

5) Adding the shorted stub of length d_2 in parallel at B completes our path on the S. C. from B to C (HOME!).

http://www.amanogawa.com/archive/transmissionA.html
- Go down to *narrow-band impedance matching*
- Pay attention to frequency ranges of operation
- Check out broad(er)-band, like tapered lines, etc.





1) Normalize load: $z_L = \dfrac{300 + j200}{100} = 3 + j2$

2) Rotate CW on the $|\Gamma| = 0.63$ ($VSWR = 4.4$) circle to intersect with the horizontal ($x = 0$) midline where $z_B = 4.4 + j0 \Rightarrow z_B = (4.4)(100\,\Omega) = 440\,\Omega$

Note: $z_B = \dfrac{1+\Gamma}{1-\Gamma} = \dfrac{1+|\Gamma|}{1-|\Gamma|} = VSWR = 4.4$ here!

Find length: $d_1 = z_B - z_A = (0.250 - 0.224)\lambda = 0.026\lambda$

3) Find $Z(\lambda/4)$ for a match with 100 Ω line:

$Z(\lambda/4) = \sqrt{(440\,\Omega)(100\,\Omega)} = 210\,\Omega$

then re-normalize on the QWT to get $z_{B'} = \dfrac{z_B}{z(\lambda/4)} = \dfrac{440\,\Omega}{210\,\Omega} = 2.09$

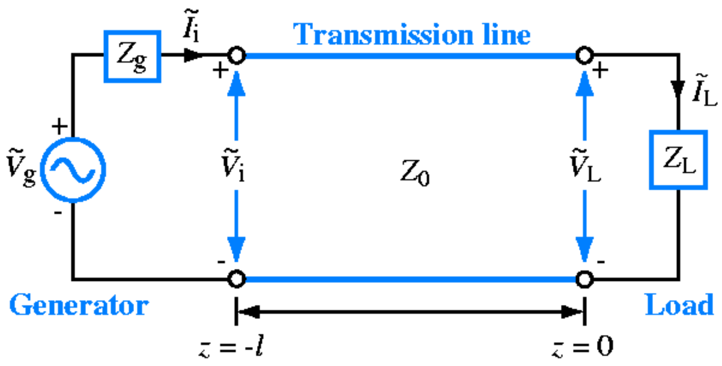
4) Rotate CW on the $|\Gamma| = 0.36$ ($VSWR = 2.09$) circle halfway ($\lambda/4$) around the chart to input of the QWT where $z_C = 0.48 + j0$ and $Z_C = (0.48)(210\,\Omega) = 100.8\,\Omega$

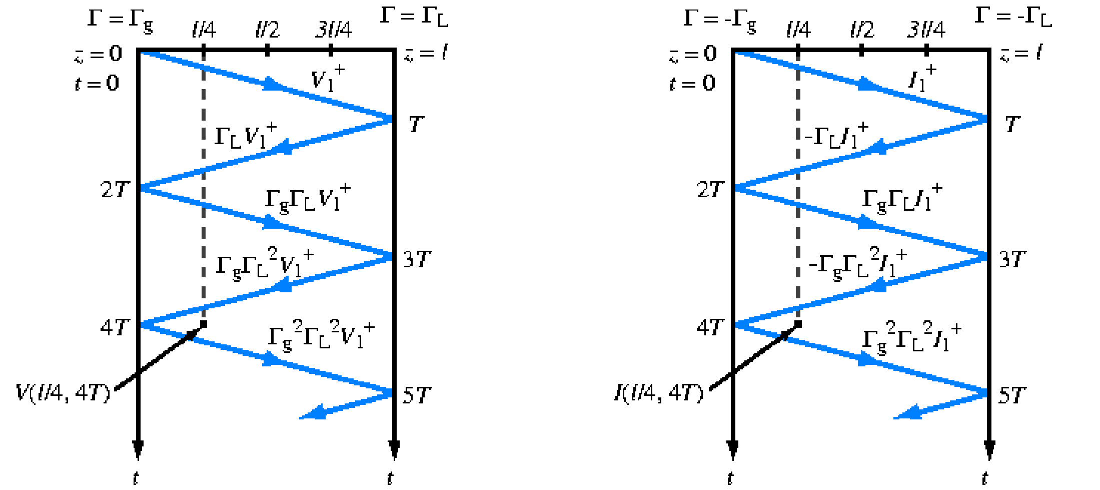
5) Now re-normalize on the $Z_0 = R_0 = 100\,\Omega$ line to get $z_D = 1.0 = VSWR$ at D (we have arrived at HOME for a match!)

http://www.amanogawa.com/archive/transmissionA.html

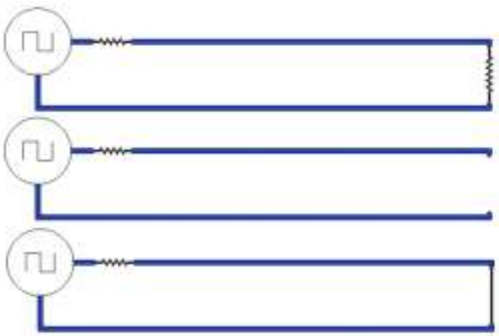
- Go down to *narrow-band impedance matching*
- Pay attention to frequency ranges of operation
- Check out broad(er)-band, like tapered lines, etc.

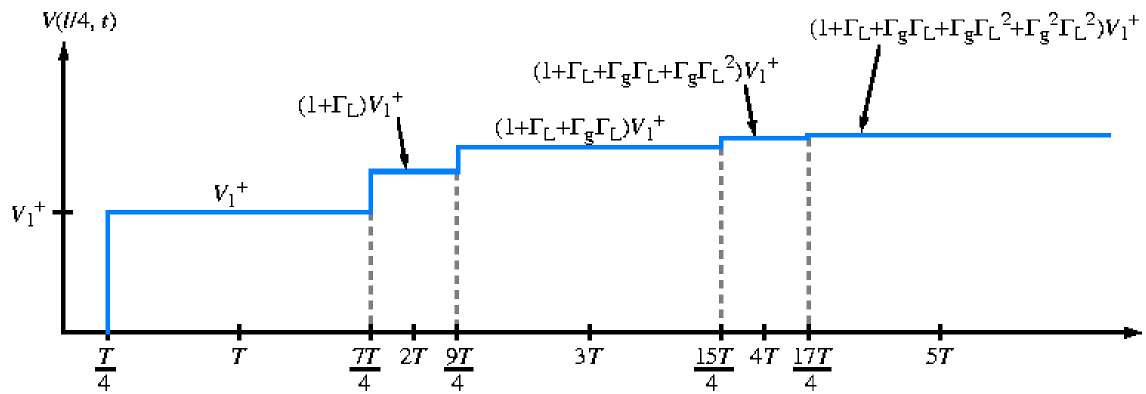
Transients (on T-Lines)





(a) Voltage bounce diagram (b) Current bounce diagram

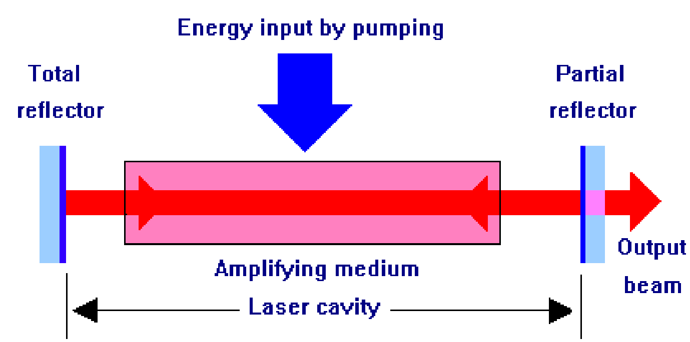




http://www.amanogawa.com/archive/signalintegrityA.html

(c) Voltage *versus* time at $z = l/4$

Resonance



Example 2-15: Pulse Propagation

The transmission-line circuit of Fig. 2-43(a) is excited by a rectangular pulse of duration $\tau = 1$ ns that starts at $t = 0$. Establish the waveform of the voltage response at the load, given that the pulse amplitude is 5 V, the phase velocity is c, and the length of the line is 0.6 m.

Solution: The one-way propagation time is

$$T = \frac{l}{c} = \frac{0.6}{3 \times 10^8} = 2 \text{ ns}.$$

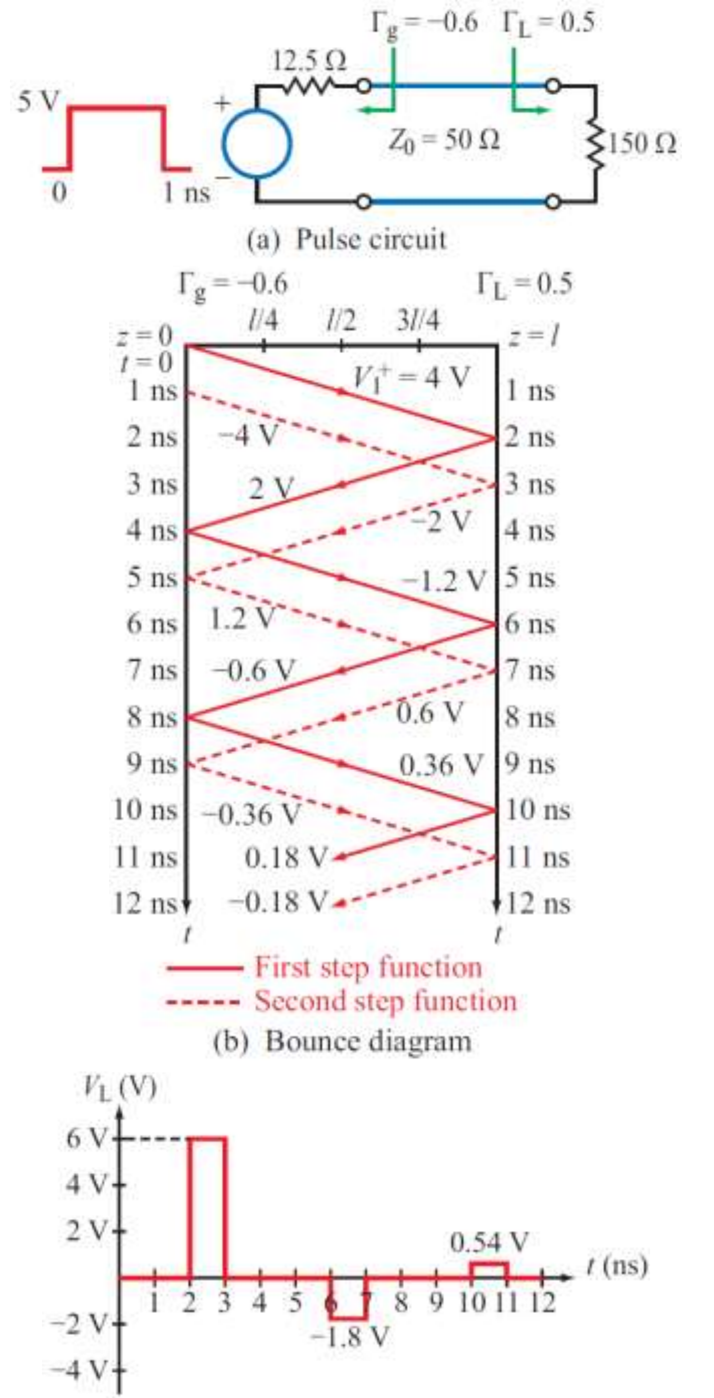
The reflection coefficients at the load and the sending end are

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = \frac{150 - 50}{150 + 50} = 0.5,$$

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{12.5 - 50}{12.5 + 50} = -0.6.$$

By Eq. (2.147), the pulse is treated as the sum of two step functions, one that starts at $t = 0$ with an amplitude $V_{10} = 5$ V and a second one that starts at $t = 1$ ns with an amplitude $V_{20} = -5$ V. Except for the time delay of 1 ns and the sign reversal of all voltage values, the two step functions will generate identical bounce diagrams, as shown in Fig. 2-43(b). For the first step function, the initial voltage is given by

$$V_1^+ = \frac{V_{01} Z_0}{R_g + Z_0} = \frac{5 \times 50}{12.5 + 50} = 4 \text{ V}.$$

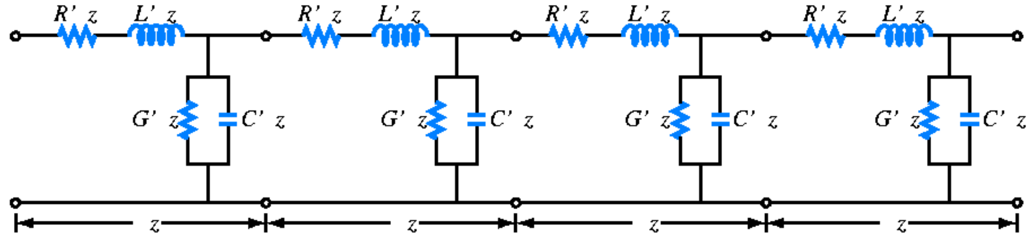


(a) Pulse circuit

(b) Bounce diagram

(c) Voltage waveform at the load

Flashback: **<u>Lossy</u> Transmission Lines**



$$\tilde{V}(z) = V_0^+ e^{-\alpha z} e^{-j\beta z} \Rightarrow v(z,t) = V_0^+ e^{-\alpha z} \cos(\omega t - \beta z)$$

$$\gamma \equiv \sqrt{(R' + j\omega L')(G' + j\omega C')} = \alpha + j\beta$$

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

Note: Lossless was $R' = G' = 0$

$\Rightarrow \alpha = 0$, $Z_0 \in$ Reals

Another Special Case Lossy Line, but $\Rightarrow \frac{R'}{L'} = \frac{G'}{C'}$ ← You design this! (as engineer)

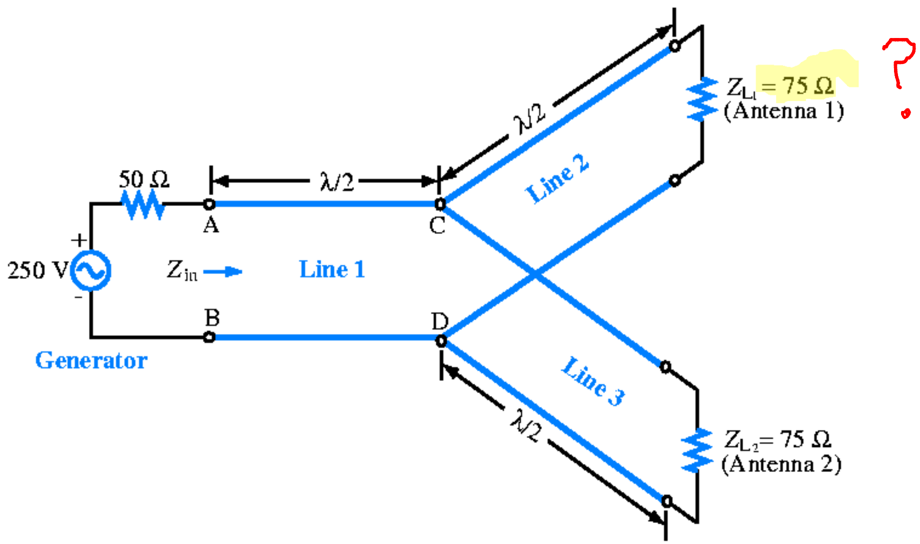
$\Rightarrow \alpha = R'\sqrt{\frac{C'}{L'}}$ and $Z_0 = \sqrt{\frac{L'}{C'}}$

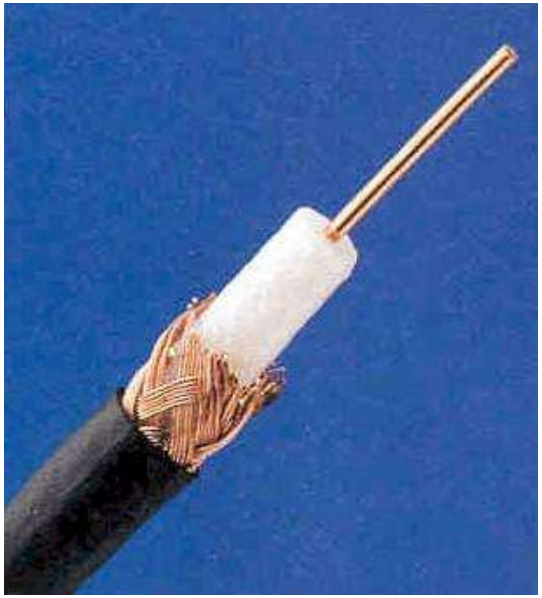
$\beta = \omega\sqrt{LC}$

$\Rightarrow u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}$

So, for this special case, α, u_p & Z_0 are not functions of frequency!

$\Rightarrow$ Distortionless Line





* Why coax vs. 2 wire???

* What about multiple wire, various length, single ground plane???

* Scalar vs. Vectors

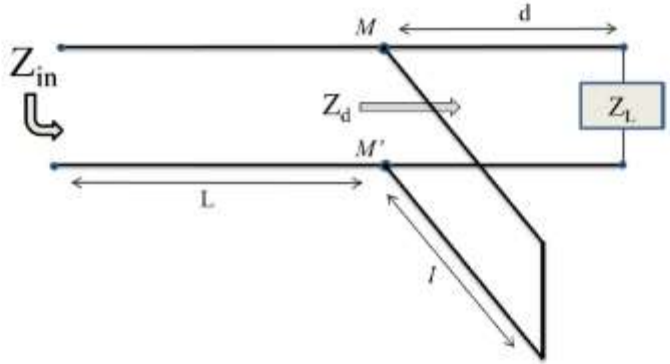
…leads into next section (chapter); Vector Analysis!!! ☺

2) The phasor voltage on a transmission line is found to be $Ae^{\gamma z}$, where γ is the (complex) propagation constant. What is the voltage in the time domain?

3) A given shorted stub behaves like an open circuit at 600 MHz. What is the closest frequency at which it behaves like a short (assuming constant phase speed)?

3) Given a lossless transmission line of known intrinsic impedance, phase speed and length, can you determine the load impedance from measurements of the input impedance? Explain.

4) (20 pts.) A given antenna can be represented by an equivalent load impedance of 100 - j50 Ω. We need to design a matching network in the configuration shown using lossless T-line (Z_0=50Ω) The wavelength is known to be 100cm.



a) What is the frequency of operation? If you can't specify, can you give me an upper or lower limit?

f ____________________

b) Given d=12.5 cm, find the impedance looking just into that d-lengthed piece of line terminated by Z_L.

Z_d = ____________________

c) Given Z_d, find the shortest length (l) of shorted stub that cancels out the reactance at point M.

l = ____________________

d) Assuming your matching efforts all work, what is the input impedance (Z_{in}) seen by the driving network?

Z_{in} = ____________________